Because human cells, including nerve cells, contain ions that can move, they are electrically conductive. For some time, scientists have believed that the electrical nature of the brain could be used to help treat certain diseases. However, the skull is a fairly good electrical insulator. Until recently, the only options for electrical stimulation of the brain were either to apply a very high potential difference across points on the skull or to surgically implant electrodes into the brain. Now there is a promising new way to study and alter the electric activity of the brain noninvasively. This technology, called transcranial magnetic stimulation (TMS), may help treat mood disorders, Parkinson’s disease, and Huntington’s disease. TMS studies have also

Be sure you know how to:
- Find the direction of the $\vec{B}$ field produced by an electric current (Section 17.2).
- Find the direction of the magnetic force exerted on moving electric charges (Sections 17.3 and 17.4).
- Explain how an electric field produces a current in a wire and how that current relates to the resistance of the wire (Sections 16.1 and 16.4).
Electromagnetic induction helped medical researchers understand the processes involved in neural repair, learning, and memory.

TMS treatment is fairly simple: a clinician places a small coil of wire on or near the patient’s scalp. The changing current through this coil produces an abrupt electric current in the brain directly under the coil, even though there is no electrical connection between the outside coil and the brain. How is this possible?

In the last chapter, we learned that an electric current through a wire produces a magnetic field. Could the reverse happen? Could a magnetic field produce a current? It took scientists many years to answer this question. In this chapter, we will investigate the conditions under which this can happen.

18.1 Inducing an electric current

In the chapter on circuits (Chapter 16), we learned that an electric current results when a battery or some other device produces an electric field in a wire. The field in turn exerts an electric force on the free electrons in the wire connected to the battery. As a result, the electrons move in a coordinated manner around the circuit—an electric current.

In this section we will learn how to produce a current in a circuit without a battery—a process called inducing a current. We start by analyzing some simple experiments in Observational Experiment Table 18.1. The experiments involve a bar magnet and a coil. The coil is not connected to a battery but is connected to a galvanometer that detects both the presence of a current and its direction. A galvanometer works like an ammeter, but it is usually not calibrated in specific units. See if you can find any patterns in the outcomes of the experiments.

### Observational Experiment Table

<table>
<thead>
<tr>
<th>Experiment 1. You hold a magnet motionless in front of a coil.</th>
<th>The galvanometer reads zero. There is no current through the coil.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram of a magnet and coil" /></td>
<td><img src="image" alt="Galvanometer reading zero" /></td>
</tr>
</tbody>
</table>
### Experiment 2.
You move the magnet toward the coil or move the coil toward the magnet.

The galvanometer needle moves to the right, indicating a current through the coil.

![Diagram of Experiment 2](image)

### Experiment 3.
You move the magnet away from the coil or move the coil away from the magnet.

The galvanometer needle moves to the left, indicating a current through the coil but opposite the direction in the last experiment.

![Diagram of Experiment 3](image)

### Experiment 4.
You turn the magnet 90° so that the poles are now perpendicular to their previous position.

The galvanometer registers a current while the magnet is turning.

![Diagram of Experiment 4](image)

### Experiment 5.
You collapse the sides of the coil together so its opening becomes very small. You pull open the sides of the collapsed coil so the area becomes large again.

In both cases, the galvanometer registers a current while the coil's area is changing, but the direction is different in each case.

![Diagram of Experiment 5](image)

### Patterns
Although no battery was used, an electric current was induced in the coil when the magnet and coil moved toward or away from each other. Current was also induced when the coil's orientation relative to the magnet or the area of the coil changed.

In Table 18.1 there was no battery, yet the galvanometer registered an electric current through a coil. For the current to exist, there must be some source of emf. What produced the emf in the experiments in Table 18.1?

Recall from our study of magnetism (in Chapter 17) that a magnetic field can exert a force on moving electrically charged particles. The force exists only if the magnetic field or a component of the field is perpendicular to the velocity of the electrically charged particles. Let’s consider again Experiment 2 in Table 18.1 in which the coil moves toward the magnet; for simplicity, we use a square loop made of conducting wire (Figure 18.1). Inside the wire there are positively charged ions that make up the lattice of the metal (and cannot leave their
locations) and negatively charged free electrons. The \( B \) field produced by the bar magnet points away from the magnet and spreads out as shown in the figure. Notice that at the top and bottom sections of the loop, a component of the \( B \) field is perpendicular to the velocity of the loop as it moves toward the right. As a result, the magnetic field exerts a force on each electron in the wire. This force causes the electrons in the wire to accelerate clockwise around the loop as viewed from the magnet (use the right-hand rule for the magnetic force on the negatively charged electrons, as discussed in Section 17.4). Note also that electrons in the vertical section of the loop closest to us accelerate upward, while the electrons in the vertical section farthest from us accelerate downward. The overall result is that due to the relative motion between the loop and the bar magnet, the electrons start moving around the loop in a coordinated fashion—we have an induced electric current.

Thus, it seems that the currents produced in Table 18.1 can be explained using the concept of the magnetic force we have previously developed (in Chapter 17). Does magnetic force also explain how transcranial magnetic stimulation (TMS) works? It does not. In the TMS procedure, the coil is not moving relative to the brain, whereas the magnetic force-based explanation requires motion of the loop relative to the magnetic field.

Perhaps there is another explanation. Let’s examine Table 18.1 from a different perspective, one that focuses on the \( B \) field itself rather than on any sort of motion. When the magnet or coil moved or rotated with respect to the magnetic field, the number of \( B \) field lines going through the area of the coil increased or decreased (Figure 18.2a). The number of \( B \) field lines through the coil also changed when the area of the coil changed (Figure 18.2b). Thus, an alternative explanation for the pattern we observed in the experiments is that when the number of \( B \) field lines through the coil’s area changes, there is a corresponding emf produced in the coil, which leads to the induced current.

We can summarize these two explanations for the induced current as follows:

**Explanation 1:** The induced current is caused by the magnetic force exerted on moving electrically charged objects (for example, the free electrons in conducting wires that are moving relative to the magnet).

**Explanation 2:** Any process that changes the number of \( B \) field lines through a coil’s area induces a current in the coil. The mechanism explaining how it happens is unclear at this point.

Let’s test these explanations with an experiment involving a change in the number of \( B \) field lines through a coil’s area, but with no relative motion. Recall that we can create a magnetic field using either a wire that carries current or a permanent magnet (Chapter 17). The current-carrying coil in Figure 18.3a has a \( B \) field that resembles the \( B \) field of the bar magnet in Figure 18.3b.
Testing Experiment Table 18.2 uses two coils, such as those shown in Figure 18.4. Coil 1 on the bottom is connected to a battery and has a switch to turn the current through coil 1 on and off. When the switch is open, there is no current in coil 1. When the switch is closed, the current in coil 1 produces a magnetic field whose field lines pass through coil 2’s area. For each of the experiments we will use the two explanations to predict whether or not there should be an induced electric current in coil 2.

![Diagram of coils and switch](image)

**Figure 18.4** Coil 1 is positioned directly under coil 2. Can a magnetic field from coil 1 induce a current in coil 2?

### Table 18.2 Testing the explanations for induced current.

<table>
<thead>
<tr>
<th>Testing experiment</th>
<th>Will a current be induced in coil 2?</th>
<th>Will a current be induced in coil 2?</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experiment 1.</strong> The switch in the circuit for coil 1 is open. There is no current in coil 1. Is there any current in coil 2?</td>
<td>There is no current in coil 1, thus there is no magnetic field at coil 2. Neither coil is moving. No current will be induced in coil 2.</td>
<td>There is no current in coil 1; therefore, there is no change in the number of ( B ) field lines through coil 2’s area. No current will be induced in coil 2.</td>
<td>The galvanometer registers no current in coil 2.</td>
</tr>
<tr>
<td><strong>Experiment 2.</strong> You close the switch in the circuit for coil 1. While the switch is being closed, the current in coil 1 increases rapidly from zero to a steady final value. Is there any current in coil 2 while the switch is being closed?</td>
<td>Neither coil is moving, thus no current will be induced in coil 2.</td>
<td>The increasing current in coil 1 produces an increasing ( B ) field. This changes the number of ( B ) field lines through coil 2’s area. A brief current should be induced in coil 2.</td>
<td>Just as the switch closes, the galvanometer needle briefly moves to the left and then returns to vertical, indicating a brief induced current in coil 2.</td>
</tr>
<tr>
<td><strong>Experiment 3.</strong> You keep the switch in the circuit for coil 1 closed. The current in coil 1 has a steady value. Is there current in coil 2?</td>
<td>Neither coil is moving. Thus no current will be induced in coil 2.</td>
<td>There is a steady current in coil 1, which results in a steady ( B ) field. Thus, the number of ( B ) field lines through coil 2’s area is not changing. Therefore, no current will be induced in coil 2.</td>
<td>The galvanometer registers no current in coil 2.</td>
</tr>
</tbody>
</table>

(continued)
Experiment 4. You open the switch again. Is there any current in coil 2 while the switch is being opened?

Neither coil is moving. Thus no current will be induced in coil 2.

The decreasing current in coil 1 produces a decreasing $B$ field. This changes the number of $B$ field lines through coil 2’s area, which should induce a brief current in coil 2.

Just as the switch opens, the galvanometer needle briefly moves to the right (opposite the direction in experiment 2), then returns to the vertical, indicating a brief induced current in coil 2.

Conclusion

The predictions based on Explanation 2 matched the outcomes in all four experiments. The predictions based on Explanation 1 did not match the outcomes in two of the four experiments.

Motion is not necessary to have an induced current. In contrast, when the number of $B$ field lines through a coil’s area changes, there is an induced current in that coil. Explanation 1 has been found not to be generally valid, but Explanation 2 has been found to be generally valid.

We have learned that it is possible to have a current in a closed loop of wire without using a battery. This phenomenon of inducing a current using a changing $B$ field is called electromagnetic induction.

Electromagnetic induction explains how transcranial magnetic stimulation (TMS) works. When the physician closes the switch in the circuit with the coil that rests on the outside of the skull, the increasing current in the coil produces a changing $B$ field within the brain, which in turn induces current in the brain’s electrically conductive tissue. A current is briefly and noninvasively induced in a small region of the brain.

**Conceptual Exercise 18.1 Moving loops in a steady magnetic field**

The figure shows four wire loops moving at constant velocity $\vec{v}$ relative to a region with a constant $B$ field (within the dashed lines). In which of these loops will electric currents be induced?

**Sketch and translate** An electric current will be induced in a loop whenever the number of $B$ field lines through the loop’s area changes.

**Simplify and diagram** For each case, current should be as follows:

1. The loop is moving into the field region, so the number of $B$ field lines through its area is increasing. A current will be induced in the loop.
2. A current will be induced in the loop for the same reasons as (1).
3. The loop is completely within the field region, thus the number of $B$ field lines is not changing. As a result, no current is induced.
4. No current is induced for the same reason as in (3).

**Try it yourself:** What happens to loops 3 and 4 as they are leaving the magnetic field region?

**Answer:** The number of $B$ field lines through each loop will change and a brief current will be induced as the loops leave the $B$ field region.

**Discovery of electromagnetic induction**

We observed electromagnetic induction with relative ease. However, the first observation of this phenomenon in 1831 was more difficult. Following Hans Oersted’s discovery in 1820 that an electric current produces a magnetic force...
on a compass needle, scientists wondered if the reverse might occur—could an object with magnetic properties produce an electric current? In 1821, British experimentalist Michael Faraday began investigating the possibility. Two difficulties lay before him—one conceptual and one technical. The conceptual difficulty was that although a steady current always produces a magnetic field, a steady magnetic field does not always produce an electric current. The technical difficulty was that the galvanometers of the time could not detect the weak currents induced in a single loop. Coils of wire (multiple loops of wire) were needed to amplify the magnetic effect. However, individual wires cannot be in contact with each other; they must be insulated from touching each other. At the time, no process existed for producing insulated wires; thus there was no way to make coils.

The technical problem was solved when American physicist Joseph Henry devised and published a method for insulating wires by wrapping them in silk. Henry was the first to observe a current being induced in a coil. He did not immediately publish his discovery, however. In 1831, upon learning about Henry’s work, Faraday used Henry’s insulation method to induce current in coils in his laboratory. Since then, scientists and engineers have devised many practical devices based on electromagnetic induction, such as electric generators, magnetic credit card readers, the transformers needed for modern electric power grids, and the pick-up coils for string and percussion instruments.

Applying electromagnetic induction: Dynamic microphones and seismometers

A dynamic microphone that converts sound vibrations into electrical oscillations employs the principle of electromagnetic induction (Figure 18.5). When a sound wave, such as a singer’s voice, strikes the diaphragm inside the microphone, the diaphragm oscillates. This oscillation moves a coil of wire attached to the diaphragm alternately closer and farther from a magnet in the microphone, corresponding to locations with stronger and weaker magnetic field. This changing magnetic field through the coil induces a changing current in the coil. The changes in the current mirror the sound waves that led to the production of this current; thus this current can then be used to store the details of the sound electronically.

A seismometer operates by the same principle. A seismometer is a sensor that detects seismic waves during an earthquake. The seismometer has a massive base with a magnet that vibrates as seismic waves pass. At the top, a coil attached to a block hangs at the end of a spring (Figure 18.6). The spring acts as a shock absorber that reduces the vibrations of the hanging block and coil relative to the base. The motion of the base relative to the coil induces a current through the coil, which produces a signal that is recorded on a seismograph.

Review Question 18.1 Your friend thinks that relative motion of a coil and a magnet is absolutely necessary to induce current in a coil that is not connected to a battery. Support your friend’s point of view with a physics argument. Then provide a counterargument and describe an experiment you could perform to disprove your friend’s idea.

18.2 Magnetic flux

In the last section we found that an electric current is induced in a coil when the number of $\vec{B}$ field lines through the coil’s area changes. This occurred when

- the strength of the $\vec{B}$ field in the vicinity of the coil changed, or
- the area $A$ of the coil changed, or
- the orientation of the $\vec{B}$ field relative to the coil changed.
In this section we will construct a physical quantity for the number of \( \vec{B} \) field lines through a coil’s area. Based on the analysis in the last section, changes in that quantity should cause an induced electric current in the coil. Physicists call this physical quantity magnetic flux \( \Phi \).

We have already defined the magnetic flux qualitatively as the number of \( \vec{B} \) field lines passing through a particular two-dimensional area \( A \). The greater the magnitude of the \( \vec{B} \) field passing through the area, the greater the number of field lines through the area. Additionally, if the area itself is larger, the number of field lines through the area is greater. If we double one or the other, the number of lines through the area should double. This suggests that the magnetic flux is proportional to the magnitude of the \( \vec{B} \) field passing through the area and to the size of the area itself. Mathematically,

\[
\Phi \propto BA
\]

How do we include in the above the dependence on the orientation of the loop relative to the \( \vec{B} \) field lines? Imagine a rigid loop of wire in a region with uniform \( \vec{B} \) field. If the plane of the loop is perpendicular to the \( \vec{B} \) field lines, then a maximum number of field lines pass through the area (Figure 18.7a). If the plane of the loop is parallel to the \( \vec{B} \) field, zero field lines pass through the area (Figure 18.7b). In between these two extremes, the magnetic flux takes on intermediary values (Figure 18.7c).

To describe this relative orientation we use a line perpendicular to the plane of the loop—the black normal vector shown in Figure 18.7. The angle \( \theta \) between this vector and the \( \vec{B} \) field lines quantifies this orientation. Since the cosine of an angle is at a maximum when the angle is zero and a minimum when the angle is 90°, the magnetic flux through the area is also proportional to \( \cos \theta \). This leads to a precise definition for the magnetic flux through an area.

**Magnetic flux \( \Phi \)** The magnetic flux \( \phi \) through a region of area \( A \) is

\[
\phi = BA \cos \theta \tag{18.1}
\]

where \( B \) is the magnitude of the uniform magnetic field throughout the area and \( \theta \) is the angle between the direction of the \( \vec{B} \) field and a normal vector perpendicular to the area. The SI unit of magnetic flux is the unit of the magnetic field (the tesla T) times the unit of area (m²), or T·m². This unit is also known as the weber (Wb).

Equation (18.1) assumes that the magnetic field throughout the area is uniform and that the area is flat. For situations in which this is not the case, you first split the area into small subareas within which the \( \vec{B} \) field is approximately uniform; then add together the magnetic fluxes through each. This book will not address such cases.

In Section 18.1, we proposed that when the number of magnetic field lines through a wire loop changes, a current is induced in the loop. We can now refine that idea and say that current is induced when there is a change in the magnetic flux through the loop’s area. In other words, if the magnetic flux throughout the loop’s area is steady, no current will be induced.

**Quantitative Exercise 18.2 Flux through a book cover**

A book is positioned in a uniform 0.20-T \( \vec{B} \) field that points from left to right in the plane of the page, shown in the figures on the next page. (For simplicity, we depict the book as a rectangular loop.) Each side of the book’s cover measures 0.10 m. Determine the magnetic flux through the cover when (a) the cover is in the plane of the page (figure a), (b) the cover is perpendicular to the plane of the page and the normal vector makes a 60° angle with the \( \vec{B} \) field (figure b), and (c) the book’s cover area is perpendicular to the plane of the page and the normal vector points toward the top of the page (figure c).
18.3 Direction of the induced current

Recall that in the experiments in Section 18.1, the galvanometer registered current in one direction for some of the experiments and in the opposite direction for others. We discovered the conditions under which a current can be induced in a wire loop, but can we also explain the direction of this current? That is the goal of this section.

Figure 18.8 shows the results of two experiments in which the number of \( \vec{B} \) field lines through a wire coil’s area is changing. As the bar magnet moves toward the coil, \( \vec{B}_{\text{ex}} \) increases and a current is induced. As the magnet moves away from the coil, \( \vec{B}_{\text{ex}} \) decreases.

**Review Question 18.2** You have a bar magnet and a gold ring. How should you position the ring relative to the magnet so that the magnetic flux through the circular area inside the ring is zero?

**Active Learning Guide**

We can evaluate these results by comparing the calculated fluxes to the number of \( \vec{B} \) field lines through the book cover’s area. Note that for the orientation of the book in (a) and (c), the \( \vec{B} \) field lines are parallel to the book’s area and therefore do not go through it. Those positions are consistent with our mathematical result. The orientation for the book in (b) is such that some \( \vec{B} \) field lines do pass through the book, which is also consistent with the nonzero mathematical result.

**Try it yourself**: A circular ring of radius 0.60 m is placed in a 0.20-T uniform \( \vec{B} \) field that points toward the top of the page. Determine the magnetic flux through the ring’s area when (a) the plane of the ring is perpendicular to the surface of the page and its normal vector points to the right and (b) the plane of the ring is perpendicular to the surface of the page and its normal vector points toward the top of the page.

**Answer**:

(a) 0; (b) 0.23 T \( \cdot \) m

\[
\Phi = (0.20 \ \text{T})(0.10 \ \text{m})^2 \cos(90^\circ) = 0
\]

(b) \( \Phi = (0.20 \ \text{T})(0.10 \ \text{m})^2 \cos(60^\circ) = 1.0 \times 10^{-3} \ \text{T} \cdot \text{m}^2 \)

(c) \( \Phi = (0.20 \ \text{T})(0.10 \ \text{m})^2 \cos(90^\circ) = 0 \)
moves closer to the coil in (a), the number of $\mathbf{B}$ field lines through the coil’s area increases (the magnetic flux through the coil’s area increases). As expected, there is a corresponding induced current. An arrow along the coil indicates the direction of this induced current as measured by a galvanometer.

Because electric currents produce a magnetic field, the induced current in the coil must also produce a magnetic field and a corresponding magnetic flux through the coil. We call this second magnetic field $\mathbf{B}_{\text{induced}}$ or $\mathbf{B}_{\text{in}}$. The direction of $\mathbf{B}_{\text{in}}$ can be determined using the right-hand rule for the $\mathbf{B}$ field. Notice that in the case shown in Figure 18.8a, the flux through the coil due to the magnet (called $\mathbf{B}_{\text{external}}$ or $\mathbf{B}_{\text{ex}}$) is increasing and the magnetic field due to the induced current $\mathbf{B}_{\text{in}}$ points in the opposite direction of $\mathbf{B}_{\text{ex}}$.

In Figure 18.8b, the bar magnet is moving away from the coil. As a result, the number of external field lines through the coil’s area (and therefore the magnetic flux through it) is decreasing. Again, there is a corresponding induced current (see Figure 18.8b). In this case, however, $\mathbf{B}_{\text{in}}$ (produced by the induced current) points in the same direction as $\mathbf{B}_{\text{ex}}$ (produced by the magnet). Can we find a pattern in these data?

Notice that in both cases $\mathbf{B}_{\text{in}}$ points in the direction that diminishes the change in the external flux through the coil. In the first experiment the flux through the coil was increasing. In that situation, $\mathbf{B}_{\text{in}}$ pointed in the opposite direction to $\mathbf{B}_{\text{ex}}$ as if to resist the increase. In the second experiment, the external flux through the coil was decreasing and $\mathbf{B}_{\text{in}}$ pointed in the same direction as $\mathbf{B}_{\text{ex}}$ as if to resist the decrease. In both situations, the $\mathbf{B}_{\text{in}}$ due to the induced current resisted the change in the external flux through the coil. Put another way, $\mathbf{B}_{\text{in}}$ points in whatever direction is necessary to try to keep the magnetic flux through the coil’s area constant.

Consider what would happen if the reverse occurred. Suppose an increasing external magnetic flux through the loop led to an induced magnetic field in the same direction as the external field. In that case, the magnetic flux due to the induced field would augment rather than reduce the total flux through the loop. This would cause an even a greater induced current, which would cause yet a greater induced magnetic field and a steeper increase in magnetic flux. In other words, just by lightly pushing a bar magnet toward a loop of wire, you would cause a runaway induced current that would continually increase until the wire melted. Of course, this would violate the conservation of energy. If such a scenario were possible, we could heat water by simply moving a bar magnet over a coil in a large tank of water.

This pattern concerning the direction of the induced current was first developed in 1833 by the Russian physicist Heinrich Lenz.

**Lenz’s law** The direction of the induced current in a coil is such that its $\mathbf{B}_{\text{in}}$ field opposes the change in the magnetic flux through the coil’s area produced by other objects. If the magnetic flux through the coil is increasing, the direction of the induced current’s $\mathbf{B}_{\text{in}}$ field leads to a decrease in the flux. If the magnetic flux through the coil is decreasing, the direction of the induced current’s $\mathbf{B}_{\text{in}}$ field leads to an increase in the flux.

Lenz’s law lets us determine the direction of the induced current’s magnetic field $\mathbf{B}_{\text{in}}$. From there we can use the right-hand rule for the $\mathbf{B}$ field to determine the direction of the induced current itself. This process is summarized in the Reasoning Skill box, which shows how to determine the direction of the induced current in a loop of wire. In this skill box, the loop of wire is in a decreasing external field $\mathbf{B}_{\text{ex}}$ that is perpendicular to the plane of the loop.
18.3 Direction of the induced current

**REASONING SKILL** Determine the direction of an induced current

The magnetic flux through a loop or coil can change because of a change in the external magnetic field, a change in the area of a loop or coil, or a change in its orientation. Because of a flux change, a current is induced in a direction that can be determined as shown below.

1. Determine the initial external magnetic flux $\Phi_{\text{ex}}$ through the coil (represented below by the number and direction of the external magnetic field lines).

2. Determine the final external magnetic flux $\Phi_{\text{ex}}$ through the coil (represented below by the smaller number of external magnetic field lines).

3. The induced flux opposes the change in the external flux (a decrease in upward external flux in this case; $\vec{B}_{\text{ex}}$ points up so that the net upward flux does not decrease).

4. Use the right-hand rule for the magnetic field to determine the direction of the induced current that will produce the $\vec{B}_{\text{in}}$ and the induced flux.

---

**CONCEPTUAL EXERCISE 18.3 Practice using Lenz’s law**

A loop in the plane of the page is being pulled to the right at constant velocity out of a region of a uniform magnetic field $\vec{B}_{\text{ex}}$. The field is perpendicular to the loop and points out of the paper in the region inside the rectangular dashed line (see the first figure below). Determine the direction of the induced electric current in the loop when it is halfway out of the field, as shown in the second figure below.

**Simplify and diagram** Thus, according to Lenz’s law the induced magnetic flux and induced magnetic field $\vec{B}_{\text{in}}$ should point out of the paper, as shown below, thus keeping the flux through the loop closer to the initial flux. The direction of the induced current that causes this induced magnetic field is determined using the right-hand rule for the $\vec{B}$ field; the induced electric current through the loop is counterclockwise.

**Try it yourself:** Notice that the induced current in the left side of the loop is still in the external magnetic field $\vec{B}_{\text{ex}}$ (as shown in the figure above). Determine the direction of the force that the external magnetic field $\vec{B}_{\text{ex}}$ exerts on the induced current in the left side of the loop.

**Answer:** To the left, opposite the direction of the loop’s velocity.
Eddy currents: an application of Lenz’s law

Conceptual Exercise 18.3 shows an example of a phenomenon called an eddy current. An eddy current usually occurs when a piece of metal moves through a magnetic field. If you were to hold a sheet of aluminum or copper between the poles of a strong horseshoe magnet, you would find that neither of the poles attracts the sheet. However, when you move the sheet out from between the poles of the magnet, especially if you pull it quickly, you encounter resistance. This force is similar to the force exerted on the loop leaving the magnetic field region in the Try It Yourself part of Conceptual Exercise 18.3.

Let’s examine this phenomenon. Figure 18.9a shows a metal sheet between the poles of an electromagnet. Pulling the sheet to the right decreases the external magnetic flux through area 1. This is similar to the situation in Conceptual Exercise 18.3, although in that exercise the external \( B \) field was out of the page instead of down. This decrease in flux induces an eddy current in the metal sheet around area 1, which circles or curls clockwise in this region (see Figure 18.9b) and according to Lenz’s law produces an induced field that points in the same direction as the external \( B \) field.

At the same time, area 2 of the sheet is entering the magnetic field region, so the magnetic flux through that area is increasing. This change in magnetic flux induces a counterclockwise eddy current.

So, how do we explain the force that points opposite the direction of the sheet’s motion? The left side of the eddy current in area 1, shown in Figure 18.9b, is still in the magnetic field region. Using the right-hand rule for the magnetic force, we find that the force exerted by the magnet on the left side of the eddy current in area 1 points toward the left when the sheet is pulled to the right, in agreement with what was observed. Similarly, the magnet also exerts a force on the part of the eddy current in area 2 that is in the magnetic field region. The right-hand rule for the magnetic force determines that this force points to the left as well. Both of these forces point opposite the direction the sheet is moving, acting as a sort of “braking” force.

What would happen if you were to push the sheet to the left instead of pulling it to the right? Using Lenz’s law, we find that the eddy currents reverse direction, and the magnetic forces exerted on them also reverse direction, again resulting in a magnetic braking effect.

Many technological applications rely on the phenomenon of resistance to the motion of a nonmagnetic metal material through a magnetic field. For instance, the braking system used in some cars, trains, and amusement park car rides consists of strong electromagnets with poles on either side of the turning metal wheels of the vehicle (Figure 18.10). When the electromagnet is turned on, magnetic forces are exerted by the electromagnet on the eddy currents in the
wheels and oppose their motion. As the turning rate of the wheels decreases, the eddy currents decrease, and therefore the braking forces decrease.

Coin sorters in vending machines also rely on magnetic braking. The coins roll down a track and through a magnetic field, which induces eddy currents that slow each coin to a speed that is based on the coin’s metal type and size. The coins exit the field region at different speeds and fly like projectiles into a bin that is specific for each type of coin. Sensors detect where the coins land to determine how much the customer paid.

Review Question 18.3 What difficulty would occur if the $\vec{B}$ field produced by the induced current enhanced the change in the external field rather than opposed the change? Give a specific example.

18.4 Faraday’s law of electromagnetic induction

In the first two sections of this chapter, we found that when the magnetic flux through a coil’s area changes, there is an induced electric current in the coil. The flux depends on the magnitude of the $\vec{B}$ field, the area of the coil, and the orientation of the coil relative to the $\vec{B}$ field. Our next goal is to construct a quantitative version of this idea that will allow us to predict the magnitude of the induced current through a particular coil. We begin with Observational Experiment Table 18.3, in which we will determine what factors affect the magnitude of the induced current produced by the flux change.

<table>
<thead>
<tr>
<th>Observational experiment</th>
<th>Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experiment 1.</strong> Rapidly move a magnet toward a coil and observe the galvanometer needle. Repeat the process, only move the magnet slowly.</td>
<td>The galvanometer registers a larger induced current when the magnet moves rapidly toward the coil compared with when it moves slowly.</td>
</tr>
<tr>
<td><strong>Experiment 2.</strong> Rotate a magnet rapidly in front of a stationary coil. Repeat the process, only rotate the magnet slowly.</td>
<td>The galvanometer registers a larger induced current when the magnet rotates rapidly compared with when it rotates slowly.</td>
</tr>
</tbody>
</table>
Chapter 18  Electromagnetic Induction

Observational experiment

Experiment 3. Use two coils, each with a different number of turns. Move the magnet toward the coil with the greater number of turns. Then move the magnet at the same speed toward the coil with fewer turns.

Patterns

- The speed at which the magnet moved or rotated affected the induced current. The shorter the time interval for the change of the flux through the coil, the greater the induced current.
- The induced current is greater in a coil with a larger number of turns than in a coil with a smaller number of turns.

In the experiments in Observational Experiment Table 18.3, we found that the induced current was greater if the same change in magnetic flux through a coil occurred in a shorter time interval $\Delta t$. Additionally, the induced current through the coil was greater in a coil with a larger number of turns $N$.

**Faraday's law of electromagnetic induction**

Quantitatively, what is the relationship between the magnitude of the induced current in a coil and the change in magnetic flux through that coil’s area? Let’s connect a single circular loop with zero resistance in series with a 100-Ω resistor (Figure 18.11a). The loop is placed between the poles of an electromagnet with its surface perpendicular to the magnetic field. An ammeter (not shown) measures the current in the loop and resistor. The upward magnetic field (up is defined as the positive direction) decreases steadily for 2.0 s (we decrease the magnitude of the $B$ field by decreasing the current in the coils of the electromagnet; note that this is not the induced current), after which the field and flux remain constant at a smaller positive value. The magnetic flux-versus-time and the induced current-versus-time graphs are shown in Figure 18.11b.

When the flux is changing at a constant rate, the current through the loop and resistor has a constant value. For example, for the first 2.0 s the slope of the magnetic flux-versus-time graph has a constant negative value of $-1.0$ T·m²/s, and the induced current has a constant positive value of $+0.010$ A. If we replace the 100-Ω resistor with a 50-Ω resistor, the current-versus-time graph has the same shape but its magnitude during the first 2.0 s doubles to $+0.020$ A. Thus, the same flux change produces different size currents in the same loop depending on the resistance of the circuit. However, the product of the current and resistance for both situations has the same value:

$$(0.010 \text{ A})(100 \Omega) = (0.020 \text{ A})(50 \Omega) = 1.0 \text{ A} \cdot \Omega = 1.0 \text{ V}$$
It almost seems that there must be a 1.0-V battery in series with the resistor, but there isn’t. The changing flux through the loop caused the induced current. However, the unit of the slope of the flux-versus-time graph is not the ampere, but the volt:

\[
\frac{\Delta \Phi}{\Delta t} = \frac{-2.0 \, \text{T} \cdot \text{m}^2}{2.0 \, \text{s}} = -1.0 \left( \frac{\text{m}^2}{\text{s}} \right) s = -1.0 \frac{\text{N} \cdot \text{m}}{\text{C}} = -1.0 \frac{\text{J}}{\text{C}} = -1.0 \text{ V}
\]

(We used the expression for the magnetic force \( F_{\text{magnetic}} = qvB \) or \( B = F_{\text{magnetic}}/qv \) to convert the magnetic field in tesla (T) to other units.) Thus the induced current is a consequence of the emf induced in the coil. The changing flux acts as a “battery” that produces a 1.0-V emf that causes the electric current. You could apply Kirchhoff’s loop rule to the circuit shown in Figure 18.11a. There is a +1.0-V potential change across the loop and a −1.0-V potential change across the resistor.

The magnitude of the emf in the loop depends only on the rate of change of flux through the loop (the slope of the flux-versus-time graph \( e_{\text{in}} = |\Delta \Phi/\Delta t| \)). When we repeat this experiment for a coil with \( N \) loops, we find that the magnitude of the induced emf increases in proportion to the number \( N \) of loops in the coil. Thus,

\[
e_{\text{in}} = N \left| \frac{\Delta \Phi}{\Delta t} \right| = \frac{N \cdot \Phi_i - \Phi_f}{t_f - t_i} \quad (18.2)
\]

This expression is known as Faraday’s law of electromagnetic induction. However, the mathematical expression was actually developed by James Clerk Maxwell.

**Faraday’s law of electromagnetic induction** The average magnitude of the induced emf \( e_{\text{in}} \) in a coil with \( N \) loops is the magnitude of the ratio of the magnetic flux change through the loop \( \Delta \Phi \) to the time interval \( \Delta t \) during which that flux change occurred multiplied by the number \( N \) of loops:

\[
e_{\text{in}} = N \left| \frac{\Delta \Phi}{\Delta t} \right| = N \left| \frac{\Delta (BA \cos \theta)}{\Delta t} \right| \quad (18.3)
\]

The direction of the current induced by this emf is determined using Lenz’s law. A minus sign is often placed in front of the \( N \) to indicate that the induced emf opposes the change in magnetic flux. However, we will not use this notation.

Faraday realized that the phenomenon of electromagnetic induction has the same effect on devices attached to a coil as a battery does. This idea enabled him to build the first primitive electric generator that produced an induced current in a coil.

**QUANTITATIVE EXERCISE 18.4**

**Hand-powered computer**

To power computers in locations not reached by any power grid, engineers have developed hand cranks that rotate a 100-turn coil in a strong magnetic field. In a quarter-turn of the crank of one of these devices, the magnetic flux through the coil’s area changes from 0 to 0.10 T⋅m² in 0.50 s. What is the average magnitude of the emf induced in the coil during this quarter-turn?

**Represent mathematically** Faraday’s law [Eq. (18.3)] can be used to determine the average magnitude of the emf produced in the coil:

\[
e_{\text{in}} = N \left| \frac{\Delta \Phi}{\Delta t} \right| = N \left| \frac{\Phi_i - \Phi_f}{t_f - t_i} \right|
\]

(continued)
**Chapter 18  Electromagnetic Induction**

**Solve and evaluate** Inserting the appropriate values, we find:

\[ e_{in} = (100) \left[ \frac{(0.10 \, \text{T} \cdot \text{m}^2) - 0}{0.50 \, \text{s} - 0} \right] = 20 \, \text{V} \]

The magnitude of this emf is about what is required by laptop computers. However, the computer only works while the coil is turning. Laptops typically have a power requirement of about 50 watts (50 joules each second), which means considerable strength and endurance would be needed to keep the coil rotating.

**Try it yourself:** Determine the average emf produced in the coil if you turn the coil one quarter-turn in 1.0 s instead of in 0.50 s.

*Answer:* 10 V.

---

**Review Question 18.4** Why do we write the law of electromagnetic induction in terms of emf rather than in terms of induced current?

**18.5 Skills for analyzing processes involving electromagnetic induction**

Faraday’s law enables us to design and understand practical applications of electromagnetic induction. For example, to design an automobile ignition system that uses spark plugs, engineers must estimate how quickly the magnetic field through a coil must be reduced to zero to produce a large enough emf to ignite a spark plug. An engineer designing an electric generator will be interested in the rate at which the generator coil must turn relative to the \( B \) field to produce the desired induced emf. The general strategy for analyzing questions like these is described and illustrated in Example 18.5.

---

**PROBLEM-SOLVING STRATEGY** Problems involving electromagnetic induction

**EXAMPLE 18.5** Determine the \( \vec{B} \) field produced by an electromagnet

To determine the \( \vec{B} \) field produced by an electromagnet, you use a 30-turn circular coil of radius 0.10 m (\( 30 \, \Omega \) resistance) that rests between the poles of the magnet and is connected to an ammeter. When the electromagnet is switched off, the \( \vec{B} \) field decreases to zero in 1.5 s. During this 1.5 s the ammeter measures a constant current of 180 mA. How can you use this information to determine the initial \( \vec{B} \) field produced by the electromagnet?

**Sketch and translate**

- Create a labeled sketch of the process described in the problem. Show the initial and final situations to indicate the change in magnetic flux.
- Determine which physical quantity is changing (\( B, A, \) or \( \theta \)), thus causing the magnetic flux to change.

![Diagram of a 30-turn coil with radius 0.10 m, a current of 0.18 A, and a time interval of 1.5 s showing the change in magnetic flux.](image)

The changing quantity (from time 0.0 s to 1.5 s) is the magnitude of the \( \vec{B} \) field produced by the electromagnet. Due to this change, the flux...
In the following three sections, we consider practical applications of electromagnetic induction that involve a change in (1) the magnitude of the \( B \) field, (2) the area of the loop or coil, or (3) the orientation of the coil relative to the \( B \) field. All of these processes involve the same basic idea: a changing magnetic flux through the area of a coil or single loop is accompanied by an induced emf around the coil or loop. In turn, this emf induces an electric current in the coil or loop.

**Review Question 18.5** In the last example, why did we assume that the \( B \) field was uniform?

**Simplify and diagram**
- Decide what assumptions you are making: Does the flux change at a constant rate? Is the magnetic field uniform?
- If useful, draw a graph of the flux and the corresponding induced emf-versus-clock reading.
- If needed, use Lenz’s law to determine the direction of the induced current.

**Represent mathematically**
- Apply Faraday’s law and indicate the quantity (\( B, A, \) or \( \cos \theta \)) that causes a changing magnetic flux.
- If needed, use Ohm’s law and Kirchhoff’s loop rule to determine the induced current.

**Solve and evaluate**
- Use the mathematical representation to solve for the unknown quantity.
- Evaluate the results—units, magnitude, and limiting cases.

---

through the coil’s area changes; thus there is an induced current in the coil. We can use the law of electromagnetic induction to find the magnitude of the initial \( B \) field produced by the electromagnet.

Assume that
- The current in the electromagnet changes at a constant rate, thus the flux through the coil does also.
- The \( B \) field in the vicinity of the coil is uniform.
- The \( B \) field is perpendicular to the coil’s surface.

\[
e_{\text{in}} = N \frac{\Phi_1 - \Phi_1}{t_1 - t_1} = N \left| \frac{0 - B_i A \cos \theta}{\Delta t} \right|
\]

The number of turns \( N \) and the angle \( \theta \) remain constant. The magnitude of the magnetic field changes. Substitute \( e_{\text{in}} = IR \) and \( A = \pi r^2 \) and solve for \( B_i \):

\[
B_i = \frac{IR \Delta t}{N \pi r^2 \cos \theta}
\]

The plane of the coil is perpendicular to the magnetic field lines, so \( \theta = 0 \) and \( \cos 0^\circ = 1 \). Inserting the appropriate quantities:

\[
B_i = \frac{IR \Delta t}{N \pi r^2} = \frac{0.18 \text{ A} \times 30 \Omega \times 1.5 \text{ s}}{30 \times \pi \times (0.1 \text{ m})^2} = 8.6 \text{ T}
\]

This is a very strong \( B \) field but possible with modern superconducting electromagnets. Let’s check the units:

\[
\frac{A \cdot \Omega \cdot \text{s}}{m^2} = \frac{V \cdot \text{s}}{m^2} = \frac{J \cdot \text{s}}{C \cdot m^2} = \frac{N \cdot m \cdot \text{s}}{C \cdot (m/s)} = \frac{N}{C} = \text{T}
\]

The units match. As a limiting case, a coil with fewer turns would require a larger \( B \) field to induce the same current. Also, if the resistance of the circuit is larger, the same \( B \) field change induces a smaller current.

**Try it yourself:** Determine the current in the loop if the plane of the loop is parallel to the magnetic field. Everything else is the same.

**Answer:** Zero.
18.6 Changing $\vec{B}$ field magnitude and induced emf

We have learned that a current is induced in a coil (or single loop, or electrically conductive region in the case of eddy currents) when the magnetic flux through the coil’s area changes. In this section, we consider examples where the flux change is due to a change in the magnitude of the $\vec{B}$ field throughout the coil’s area. This is the case in transcranial magnetic stimulation (TMS), which we investigated qualitatively earlier in this chapter.

**Example 18.6 Transcranial magnetic stimulation**

The magnitude of the $\vec{B}$ field from a TMS coil increases from 0 T to 0.2 T in 0.002 s. The $\vec{B}$ field lines pass through the scalp into a small region of the brain, inducing a small circular current in the conductive brain tissue in the plane perpendicular to the field lines. Assume that the radius of the circular current in the brain is 0.0030 m and that the tissue in this circular region has an equivalent resistance of 0.010 $\Omega$. What are the direction and magnitude of the induced electric current around this circular region of brain tissue?

**Sketch and translate** We first sketch the situation: a small coil on the top of the scalp and a small circular disk region inside the brain tissue through which the changing magnetic field passes (see the figure below). The change in magnetic flux through this disk is caused by the increasing $\vec{B}$ field produced by the TMS coil (called $\vec{B}_{ex}$). This change in flux causes an induced emf, which produces an induced current in the brain tissue. The direction of this current can be determined using Lenz’s law.

**Represent mathematically** To find the magnitude of the induced emf, use Faraday’s law:

$$e_{in} = N \frac{\Phi_f - \Phi_i}{t_f - t_i} = NA \cos \theta \frac{B_{ex} - B_{exi}}{t_f - t_i}$$

where the magnetic flux through the loop at a specific clock reading is $\Phi = B_{ex} A \cos \theta$. The area $A$ of the loop and the orientation angle $\theta$ between the loop’s normal vector and the $\vec{B}_{ex}$ field are constant, so

$$e_{in} = N \frac{B_{ex} A \cos \theta - B_{exi} A \cos \theta}{t_f - t_i} = NA \cos \theta \frac{B_{ex} - B_{exi}}{t_f - t_i}$$

Using our understanding of electric circuits, we relate this induced emf to the resulting induced current:

$$I_{in} = \frac{e_{in}}{R}$$

**Solve and evaluate** Combine these two equations and solve for the induced current:

$$I_{in} = \frac{1}{R} e_{in} = \frac{1}{R} \left( NA \cos \theta \frac{B_{ex} - B_{exi}}{t_f - t_i} \right)$$

$$= \frac{N \pi r^2 \cos \theta}{R} \frac{B_{ex} - B_{exi}}{t_f - t_i}$$

$$= \frac{(1)(0.0030 \text{ m})^2(1)}{0.010 \Omega} \frac{0.2 \text{ T} - 0}{0.002 \text{ s} - 0} = 0.28 \text{ A}$$

Note that the normal vector to the loop’s area is parallel to the magnetic field; therefore, $\cos \theta = \cos(0^\circ) = 1$. 

Simplify and diagram Assume that $\vec{B}_{ex}$ throughout the disk of brain tissue is uniform and increases at a constant rate. Model the disk-like region as a single-turn coil. Viewed from above, $\vec{B}_{ex}$ points into the page (shown as X’s in the figure top right). Since the number of $\vec{B}$ field lines is increasing into the page, the $\vec{B}_{in}$ field produced by the induced current will point out of the page (shown as dots). Using the right-hand rule for the $\vec{B}_{in}$ field, we find that the direction of the induced current is counterclockwise.

**Top view of induced current**

- $\vec{B}_{in}$ is increasing into the page (into the head).
- $\vec{B}_{in}$ is out of the page (out of the head).
18.7 Changing area and induced emf

When located in a region with nonzero $\vec{B}$ field, a change in a coil’s area also results in a change in the magnetic flux through that area. There is then a corresponding induced emf producing an induced electric current circling the area.

**Example 18.7 Lighting a bulb**

A 10-Ω lightbulb is connected between the ends of two parallel conducting rails that are separated by 1.2 m, as shown in the figure below. A metal rod is pulled along the rails so that it moves to the right at a constant speed of 6.0 m/s. The two rails, the lightbulb, its connecting wires, and the rod make a complete rectangular loop circuit. A uniform 0.20-T magnitude $B_{ex}$ field points downward, perpendicular to the loop’s area. Determine the direction of the induced current in the loop, the magnitude of the induced emf, the magnitude of the current in the lightbulb, and the power output of the lightbulb.

**Sketch and translate** We sketch the situation as shown, top right. Choose the normal vector for the loop’s area to point upward. The loop’s area increases as the rod moves away from the bulb. Because of this, the magnitude of the magnetic flux through the loop’s area is increasing as the rod moves to the right.

**Simplify and diagram** Assume that the rails, rod, and connecting wires have zero resistance. The induced field $B_{in}$ due to the loop’s induced current should point upward, resisting the change in the downward increasing magnetic flux through the loop’s area. Using the right-hand rule for the $B_{in}$ field, we find that the direction of the induced current is counterclockwise.

**Represent mathematically** To find the magnitude of the induced emf, use Faraday’s law:

$$e_{in} = N \frac{\Phi_f - \Phi_i}{t_f - t_i}$$

The angle between the loop’s normal line and the $B_{ex}$ field is 180°. The magnitude of the $B_{ex}$ field is constant. Therefore:

$$e_{in} = (1) \frac{B_{ex} A_f \cos(180°) - B_{ex} A_i \cos(180°)}{t_f - t_i}$$

$$= B_{ex} \frac{A_f - A_i}{t_f - t_i}$$

(continued)
The area \( A \) of the loop at a particular clock reading equals the length \( L \) of the sliding rod times the \( x \)-coordinate of the sliding rod (the origin of the \( x \)-axis is placed at the bulb.) Therefore:

\[
e_{\text{in}} = B_{\text{ex}} \left[ \frac{x_f - x_i}{t_f - t_i} \right] = B_{\text{ex}} L \frac{x_f - x_i}{t_f - t_i}
\]

The quantity inside the absolute value is the \( x \)-component of the rod’s velocity, the absolute value of which is the rod’s speed \( v \). Using Ohm’s law, the induced current depends on the induced emf and the bulb resistance:

\[
I_{\text{in}} = \frac{e_{\text{in}}}{R}
\]

The power output of the light bulb will be

\[
P = I_{\text{in}}^2 R
\]

**Solve and evaluate** The magnitude of the induced emf around the loop is

\[
e_{\text{in}} = B_{\text{ex}} L v = (0.20 \, \text{T})(1.2 \, \text{m})(6.0 \, \text{m/s}) = 1.44 \, \text{V}
\]

The current in the bulb is

\[
I_{\text{in}} = \frac{1.44 \, \text{V}}{10 \, \Omega} = 0.14 \, \text{A}
\]

The lamp should glow, but just barely, since its power output is only

\[
P = I_{\text{in}}^2 R = (0.14 \, \text{A})^2(10 \, \Omega) = 0.21 \, \text{W}
\]

**Try it yourself:** Suppose the rod in the last example moves at the same speed but in the opposite direction so that the loop’s area decreases. Determine the magnitude of the induced emf, the magnitude of current in the bulb, and the direction of the current.

**Answer:** 1.4 V, 0.14 A, and clockwise.

---

**Figure 18.12** An alternative way to analyze the motion of the rod in terms of electric and magnetic forces. (a) External magnetic field exerts a force on the electrons in the moving rod. (b) The electrons accumulate on one end and leave the other end positively charged. (c) The electric and magnetic forces exerted on the electron cancel.

![Diagram](image)

---

**Limitless electric energy?**

We have just found that the induced emf depended on the speed with which a metal rod is pulled along the rails. By accelerating the rod to a speed of our choosing, we could induce whatever emf we desired. Once this speed is reached, would this method of obtaining the emf by moving a conductor in the external magnetic field maintain the current indefinitely?

The moving rod resulted in an emf and an induced current in the rails, bulb, and rod. However, if we use the right-hand rule for magnetic force, we find that the magnetic field exerts a force on the induced current in the rod toward the left, causing it to slow down. From an energy perspective, the kinetic energy of the rod is being transformed into light and thermal energy in the bulb. In order to keep the rod moving at constant speed, some other object must exert a force on it to the right doing positive work on the rod.

**Motional emf**

The emf produced in Example 18.7 is sometimes called motional emf; it is caused by the motion of an object through the region of a magnetic field. We explained this emf using the idea of electromagnetic induction. Is it possible to understand it just in terms of magnetic forces? When an electrically charged object with charge \( q \) moves within a region with nonzero \( B \) field, the field exerts a magnetic force on it (\( F_{\text{mag}} = q \mathbf{v} \times \mathbf{B} \)). Consider the system shown in **Figure 18.12a** with the rod sliding at velocity \( \mathbf{v} \) along the rails. The external magnetic field \( B_{\text{ex}} \) points into the page. Inside the rod are fixed positively charged ions and negatively charged free electrons. When the rod slides to the right, all of its charged particles move with it. According to the right-hand rule for the magnetic force, the external magnetic field exerts a magnetic force on the electrons toward end I. The positive charges cannot move inside the rod, but the free electrons can. The electrons accumulate at end I, leaving end II with a deficiency of electrons (a net positive charge). The ends of the rod become charged, as shown in **Figure 18.12b**.
These separated charges create an electric field $E$ in the rod that exerts a force of magnitude $F_{\text{ion}} = qE$ on other electrons in the rod; the electric field exerts a force on negative electrons toward II (Figure 18.12c) opposite the direction of the magnetic force. When the magnitude of the electric force equals the magnitude of the magnetic force, the accumulation of opposite electric charge at the ends of the rod ceases. Then,

$$qvB = qE \quad \text{or} \quad E = vB$$

An electric potential difference is produced between points I and II that depends on the magnitude of the electric field $E$ in the rod and the distance $L$ between ends I and II:

$$\varepsilon_{\text{motional emf}} = |\Delta V_{\text{I-II}}| = EL = vBL \quad (18.4)$$

The above expression for motional emf is the same expression we derived in Example 18.7 using Faraday’s law. Thus, for problems involving conducting objects moving in a magnetic field, we can use either Faraday’s law or the motional emf expression to determine the emf produced—either method will provide the same result.

**Review Question 18.7** Suppose the rod in Example 18.7 was one-third the length and the magnetic field was four-fifths the magnitude. How fast would the rod need to move to produce the same emf? Would the current induced in this case be the same as for Example 18.7? Explain.

### 18.8 Changing orientation and induced emf

In the previous two sections, we investigated processes where emf was induced when the magnitude of the $\vec{B}$ field changed or when the area of a loop within the $\vec{B}$ field region changed. In this section we investigate what happens when the orientation of a loop changes relative to the direction of the $\vec{B}$ field. This process has many practical applications, the most important being the electric generator.

#### The electric generator

Worldwide, we convert an average of 310 J per person of electric potential energy into other less useful energy forms every second. Electric generators make this electrical potential energy available by converting mechanical energy (such as water rushing through a hydroelectric dam) into electric potential energy.

To understand how an electric generator works, consider a very simple device that consists of a loop of wire attached to a turbine (a propeller-like object that can rotate). The loop is positioned between the poles of an electromagnet that produces a steady uniform $\vec{B}$ field. A Bunsen burner next to the turbine heats a flask of water (Figure 18.13). The water is converted to steam, which strikes the blades of the turbine, causing the turbine to rotate. The loop of wire attached to the turbine rotates in the $\vec{B}$ field region. When the loop’s surface is perpendicular to the $\vec{B}$ field, the magnetic flux through the loop’s area is at a maximum. One quarter turn later, the $\vec{B}$ field lines are parallel to the loop’s area and the flux through it is zero. After another quarter turn, the flux is again at its maximum magnitude, but negative in value since the...
Chapter 18: Electromagnetic Induction

Look for consistency among three representations for the same clock reading: (a) the position of the loop, (b) the changing flux through it, and (c) the changing emf around it (the positive direction is counterclockwise as seen in Figure 18.14).

orientation of the loop’s area is opposite what it was originally. This changing magnetic flux through the loop’s area causes a corresponding induced emf, which produces a current that changes direction each time the loop rotates one half turn. Current that periodically changes direction in this way is known as alternating current (AC).

A coal-fired power plant is based on this process. Coal is burned to heat water, converting it to steam. The high-pressure steam pushes against turbine blades, causing the turbine and an attached wire coil to rotate in a strong $\vec{B}$ field. The resulting emf drives the electric power grid.

**Emf of a generator**

How can we determine an expression for the emf produced by an electric generator? Consider the changing magnetic flux through a loop’s area as it rotates with constant rotational speed $\omega$ in a constant uniform $\vec{B}$ field (Figure 18.14). If there is an angle $\theta$ between the loop’s normal vector and the $\vec{B}$ field, then the flux through the loop’s area is

$$\Phi = B_{ex}A \cos \theta$$

Since the loop is rotating, $\theta$ is continuously changing. The loop is rotating with zero rotational acceleration ($\alpha = 0$); we can describe the motion with rotational kinematics (see Chapter 8):

$$\theta = \theta_0 + \omega t + \frac{1}{2} \alpha t^2 = \theta_0 + \omega t + \frac{1}{2}(0)t^2 = \theta_0 + \omega t$$

If we define the initial orientation $\theta_0$ to be zero, then

$$\theta = \omega t$$

where $\omega$ is the constant rotational speed of the loop. This means that the magnetic flux $\Phi$ through the loop’s area as a function of time $t$ is

$$\Phi = B_{ex}A \cos (\omega t)$$

For a side view of the rotating loop see Figure 18.15a. Figure 18.15b shows a graph of the magnetic flux through the loop’s area as a function of time.

According to Faraday’s law Eq. (18.2), the induced emf around a coil (a multi-turn loop) is

$$e_{in} = N \left| \frac{\Delta \Phi}{\Delta t} \right| = N \left| \frac{\Phi_t - \Phi_i}{t_i - t_i} \right|$$

Since $\Phi$ is continually changing, we should use calculus to write the above equation. However, in this text, we will simply show the result:

$$e_{in} = N B_{ex} A \omega \sin (\omega t) \quad (18.5)$$

where $N$ is the number of turns in a coil rotating between the poles of the magnet.

Figure 18.15c shows a graph of the induced emf as a function of clock reading. Comparing Figures 18.15b and c, you will see a pattern. The value of $e_{in}$ at a particular clock reading equals the negative value of the slope of the $\Phi$-versus-$t$ graph at that same clock reading. This makes sense, since the induced emf is related to the rate of change of the magnetic flux through the loop’s area. Slopes represent exactly that, rates of change.

Electric power plants in the United States produce an emf with frequency $f$ equal to 60 Hz (Hz is a unit of frequency; 60 Hz means the emf undergoes 60 full cycles in 1 second). This corresponds to a generator coil with a rotational speed $\omega = 2\pi f = 2\pi(60 \text{ Hz}) = 120\pi \text{ rad/s}$
These power plants can produce a peak (maximum) emf as high as 20 kV. The peak emf produced by a generator occurs when \( \sin(\omega t) = 1 \) and when \( \sin(\omega t) = -1 \). At those times,
\[
\varepsilon_{\text{in max}} = N B \varepsilon A \omega.
\] (18.6)

**QUANTITATIVE EXERCISE 18.8 Bicycle light generator**

The label on the Schmidt E6 bicycle dynamo headlight indicates that the light has a power output of 3 W and a peak emf of 6 V. The generator (also called a dynamo) for the lightbulb has a cylindrical hub that rubs against the edge of the bike tire, causing a coil inside the generator to rotate, as shown below. When the bicycle is traveling at a speed of 5.4 m/s, the coil rotates with frequency of 80 Hz (80 revolutions per second). The \( B \) field in the vicinity of the coil is uniform and has a magnitude of 0.10 T. The coil is a rectangle with dimensions 1.0 cm \( \times \) 3.0 cm. Without taking the light apart, determine how many turns there are in the generator coil.

**Represent mathematically** The number of turns in the coil is related to the maximum emf the generator can produce [Eq. (18.6)]:
\[
\varepsilon_{\text{in max}} = NBA \omega = NBA(2\pi f)
\]

**Solve and evaluate** Solving for \( N \) and inserting the appropriate values:
\[
N = \frac{\varepsilon_{\text{in max}}}{2\pi BA} = \frac{6.0 \text{ V}}{2\pi(80 \text{ Hz})(0.10 \text{ T})(0.01 \text{ m} \times 0.03 \text{ m})} = 400
\]

A generator coil with this number of turns is reasonable. Let’s check limiting cases. If the magnetic field, the coil area, or the frequency is larger, then fewer turns are needed for the peak emf to be 6.0 V, which is reasonable.

**Try it yourself** While riding your bike up a hill, you pedal harder; however, your bike speed reduces from 5.4 m/s to 2.7 m/s. How would these conditions affect the emf produced by the bicycle light generator in the last example?

**Answer** The peak emf would be 3.0 V, since the loop turning frequency would decrease to half the previous value.

**Review Question 18.8** How does the law of electromagnetic induction explain why there is an induced emf in a rotating generator coil?

### 18.9 Transformers: Putting it all together

Another useful application of electromagnetic induction is the transformer, a device that increases or decreases the maximum value of an alternating emf.

A transformer consists of two coils, each wrapped around an iron core (ferromagnetic) ([Figure 18.16](#)). The core confines the magnetic field produced by the electric current in one coil so that it passes through the second coil instead of spreading outside. An alternating emf across the primary coil is converted into a larger or smaller alternating emf across the secondary coil, depending on the number of loops in each coil.

Transformers are used in many electronic devices. They are also essential for transmitting electric energy from a power plant to your house. The rate of this electric energy transmission is proportional to the product of the emf across the power lines and the electric current in the lines. If the emf is low, considerable electric current is needed to transmit a considerable amount of energy. However, due to the electrical resistance of the power lines much of the electric energy is converted into thermal energy. The rate of this conversion is \( P = I^2 R \). To reduce
the $I^2R$ losses, the transmission of electric energy is done at high peak emf (about 20,000 V) and low current. Transformers then reduce this peak emf to about 170 V for use in your home. How does a transformer change the peak emf?

Suppose there is an alternating current in the primary coil, the coil connected to an external power supply. The secondary coil is connected to an electrical device, but this device requires an emf that is different from what the external power supply produces. The alternating current in the primary coil produces a $\vec{B}$ field within the transformer core. Since the current is continuously changing, the magnetic flux through the primary coil’s area is also continuously changing. Thus, an emf is induced in the primary coil:

$$e_p = N_p \frac{\Delta \Phi_p}{\Delta t}$$

where the $p$ subscript refers to the primary coil.

In an efficient transformer, nearly all of the $\vec{B}$ field lines passing through the primary coil’s area also pass through the secondary coil’s area. As a result, there is a changing magnetic flux through the secondary coil’s area as well as a corresponding emf produced in it:

$$e_s = N_s \frac{\Delta \Phi_s}{\Delta t}$$

If the transformer is perfectly efficient, then the rates of change of the magnetic flux through one turn of each coil are the same:

$$\frac{\Delta \Phi_p}{\Delta t} = \frac{\Delta \Phi_s}{\Delta t}$$

Using the results from Faraday’s law:

$$\frac{e_p}{N_p} = \frac{e_s}{N_s} \Rightarrow e_s = \frac{N_s}{N_p} e_p$$

We see that the emf in the secondary coil can be substantially larger or smaller than the emf in the primary coil depending on the number of turns in each coil. Engineers use this result to design transformers for specific purposes. For example, a step-down transformer can convert the 120-V alternating emf from a wall socket to a 9-V alternating emf, which is then converted to DC to power a laptop computer.

**QUANTITATIVE EXERCISE 18.9 Transformer for laptop**

Your laptop requires a 24-V emf to function. What should be the ratio of the primary coil turns to secondary coil turns if this transformer is to be plugged into a standard house AC outlet (effectively a 120-V emf)?

**Represent mathematically** The ratio we are looking for is related to the coil emfs by Eq. (18.7):

$$e_s = \frac{N_s}{N_p} e_p$$

**Solve and evaluate** Solving for the ratio and inserting the appropriate values:

$$\frac{N_p}{N_s} \frac{e_p}{e_s} = \frac{120 \text{ V}}{24 \text{ V}} = 5$$

The primary coil needs to have five times the number of turns as the secondary coil. This is a step-down transformer, since the resulting secondary coil peak emf is lower than the primary coil peak emf.

**Try it yourself:** If the primary coil had 200 turns, how many turns should the secondary coil have to reduce the peak emf from 120 V to 6 V?

**Answer:** 10 turns.
18.10 Mechanisms explaining electromagnetic induction

Some transformers are designed to increase rather than decrease emf. In a car that uses spark plugs for ignition, a transformer converts the 12-V potential difference of the car battery to the 20,000-V potential difference needed to produce a spark in the engine’s cylinder (Figure 18.17). The battery supplies a steady current in the transformer. An electronic switching system in the circuit can open the circuit, stopping the current in the primary coil in a fraction of a millisecond. This causes an abrupt change in the magnetic flux through the primary coil of the transformer, which leads to an induced emf in the secondary coil. The secondary coil is attached to a spark plug that has a gap between two conducting electrodes. When the potential difference across the gap becomes sufficiently high, the air between the electrodes ionizes. When the ionized atoms recombine with electrons, the energy is released in the form of light—a spark that ignites the gasoline.

The induced emf in the secondary coil is much greater than the 12 V in the primary coil for three reasons. First, the secondary coil has many more turns than the primary coil \(N_s \gg N_p\). Second, the magnetic flux through the primary coil’s area decreases very quickly (the \(\Delta t\) in the denominator of Faraday’s law is very small), resulting in a large induced emf \(e_s\) to which \(e_i\) is proportional [see Eq. (18.2)]. Third, the ferromagnetic core (usually iron) passing through the two coils significantly increases the \(\vec{B}\) field within it (Section 17.8). For these three reasons, it is possible for \(e_s \gg e_i\): the 12-V car battery can produce a 20,000-V potential difference across the electrodes of a spark plug.

Review Question 18.9 How does a transformer achieve different induced peak emfs across its primary and secondary coils?

18.10 Mechanisms explaining electromagnetic induction

Faraday’s law describes how a changing magnetic flux through a wire loop is related to an induced emf, but it does not explain how the emf comes about. In this section we will explain the origin of the induced emf.

A changing \(\vec{B}_{ex}\) field has a corresponding \(\vec{E}\) field

We know that a changing magnetic flux induces an electric current in a stationary loop (Figure 18.18). Because the loop is not moving, there is no net magnetic force exerted on the free electrons in the wire. Thus, an electric field...
must be present. The electric field that drives the current exists throughout the wire. This electric field is not produced by charge separation, but by a changing magnetic field.

If we were to represent it with \( \vec{E} \) field lines, those lines would have no beginning or end—they would form closed loops. This electric field essentially “pushes” the free electrons along the loop. We can describe it quantitatively with the emf. But this emf is very different from the emf produced by a battery. For the battery, the emf is the result of charge separation across its terminals. For the induced emf, the electric field that drives the current is everywhere in the wire. So the emf is actually distributed throughout the entire loop. You might visualize it as an electric field “gear” with its teeth hooked into the electrons in the wire loop, pushing the free electrons along the wire at every point.

What do we now know about electricity and magnetism?

We have learned a great deal about electric and magnetic phenomena. We learned about electrically charged objects that interact via electrostatic (Coulomb) forces. Stationary electrically charged objects produce electric fields, and electric field lines start on positive charges and end on negative charges (Figure 18.19a). In our study of magnetism (Chapter 17), we learned that moving electrically charged objects and permanent magnets interact via magnetic forces and produce magnetic fields. Magnetic field lines do not have beginnings or ends (Figure 18.19b), as there are no individual magnetic charges (magnetic monopoles).

When we studied electric circuits (Chapter 16) we learned that electric fields cause electrically charged particles inside metal wires to move in a coordinated way—electric currents. Later we learned that electric currents produce magnetic fields (Figure 18.19c). In this chapter, we learned about the phenomenon of electromagnetic induction and its explanation: a changing magnetic field is always accompanied by a corresponding electric field (Figure 18.19d). However, this new electric field is not produced by electric charges, and its field lines do not have beginnings or ends.

Except for the lack of magnetic charges, there is symmetry between electric and magnetic fields. This symmetry leads us to pose the following question: If in a region where the magnetic field is changing there is a corresponding electric field, is it possible that in a region where the electric field is changing there could be a corresponding magnetic field (Figure 18.19e)? This hypothesis, suggested in 1862 by James Clerk Maxwell, led to a unified theory of electricity and magnetism, a subject we will investigate in our chapter on electromagnetic waves (Chapter 24).

Review Question 18.10 Explain how (a) an electric current is produced when part of a single wire loop moves through a magnetic field and how (b) an electric current is produced when an external magnetic flux changes through a closed loop of wire.
## Summary

<table>
<thead>
<tr>
<th>Words</th>
<th>Pictorial and physical representations</th>
<th>Mathematical representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The magnetic flux through a loop’s area depends on the size of the area, the $\vec{B}$ field magnitude, and the orientation of the loop relative to the $\vec{B}$ field. (Sections 18.2, 18.4)</td>
<td><img src="image1" alt="Diagram of magnetic flux through a loop" /></td>
<td>$\Phi = BA \cos \theta$  Eq. (18.1) when $B$ is constant through the loop area</td>
</tr>
<tr>
<td><strong>Electromagnetic induction</strong> In a region with a changing magnetic field, there is a corresponding induced electric field. When a wire loop or coil is placed in this region, the magnetic flux through that loop changes and an electric current is induced in the loop. (Section 18.4)</td>
<td><img src="image2" alt="Diagram of electromagnetic induction" /></td>
<td>$e_{in} = N \frac{\Delta \Phi}{\Delta t} = N \frac{\Phi_f - \Phi_i}{t_f - t_i}$  Eq. (18.2)</td>
</tr>
<tr>
<td><strong>Lenz’s law</strong> When current is induced, its direction is such that the $\vec{B}$ field it produces opposes the change in the magnetic flux through the loop. (Section 18.3)</td>
<td><img src="image3" alt="Diagram of Lenz’s law" /></td>
<td></td>
</tr>
<tr>
<td>An electric generator produces an emf by rotating a coil within a region of strong $\vec{B}$ field, an important application of electromagnetic induction. (Section 18.8)</td>
<td><img src="image4" alt="Diagram of electric generator" /></td>
<td>$e_{in} = NB_0 A \omega \sin (\omega t)$  Eq. (18.5)</td>
</tr>
<tr>
<td><strong>Transformers</strong> are electrical devices used to increase or decrease the peak value of an alternating emf. (Section 18.9)</td>
<td><img src="image5" alt="Diagram of transformers" /></td>
<td>$e_s = \frac{N_s}{N_p} e_p$  Eq. (18.7)</td>
</tr>
</tbody>
</table>
Questions

Multiple Choice Questions

1. In which of the following experiments is electric current induced?
   I. The north pole of a strong bar magnet is held stationary in front of a coil.
   II. A strong magnet is rotating in front of the coil.
   III. A strong magnet moves from in front of the coil to above it.
   IV. The north pole of a strong magnet moves toward the coil from the side.
   V. A strong magnet rotates in the plane of the coil at its side.
   (a) I, II, IV, and V  (b) III and IV
   (c) II, III, and V  (d) II and III
   (e) I, II, IV, and V

2. If you move the coil in Figure Q18.2 toward the N pole of the large electromagnet, would an electric current be induced?

   **Figure Q18.2**

   (a) Yes  (b) No
   (c) It depends on the current in the electromagnet.

3. The magnetic flux through a loop is 10 T·m². This means that:
   (a) The magnetic field is parallel to the loop.
   (b) No current is induced.
   (c) Since the flux is not zero, current is induced.
   (d) Both (a) and (c) could be correct.

4. Your friend says that the emf induced in a coil supports the changing flux through the coil rather than opposes it. According to your friend, what happens when the magnetic flux increases slightly?
   (a) The induced current will increase continuously.
   (b) The coil will get hot and eventually melt.
   (c) The induced emf will get larger.
   (d) None of the above.
   (e) (a), (b), and (c) will occur.

5. A metal ring lies on a table. The S pole of a bar magnet moves down toward the ring from above and perpendicular to its surface. The induced current as seen from above is which of the following?
   (a) Clockwise  (b) Counterclockwise
   (c) Zero—it only changes when the N pole approaches

6. One coil is placed on top of another. The bottom coil is connected in series to a battery and a switch. With the switch closed, there is a clockwise current in the bottom coil. When the switch is opened, the current in the bottom coil decreases abruptly to zero. What is the direction of the induced current in the top coil, as seen from above while the current in the bottom coil decreases?
   (a) Clockwise  (b) Counterclockwise
   (c) Zero—the current is induced only when the coils move relative to each other
   (d) There is not enough information to answer this question.

7. Two coils are placed next to each other on the table. The coil on the right is connected in series to a battery and a switch. With the switch closed, there is a clockwise current in the right coil as seen from above. When the switch is opened, the current in the right coil decreases abruptly to zero. What is the induced current in the coil on the left as seen from above while the current in the right coil decreases?
   (a) Clockwise  (b) Counterclockwise
   (c) Zero because the current is only present when the coils move relative to each other
   (d) Zero because there is no magnetic field through the coil on the left

8. Two identical bar magnets are dropped vertically from the same height. One magnet passes through an open metal ring, and the other magnet passes through a closed metal ring. Which magnet will reach the ground first?
   (a) The magnet passing through the closed ring
   (b) The magnet passing through the open ring
   (c) The magnets arrive at the ground at the same time.
   (d) There is too little information to answer this question.

9. A window’s metal frame is essentially a metal loop through which a magnetic field can change when the window swings shut abruptly. The metal frame is 1.0 m × 0.50 m and Earth’s magnetic field makes an angle of 53° relative to the horizontal. Which answer below is closest to the average induced emf when the window swings shut?
   (a) $9.9 \times 10^{-3}$ V  (b) $7.5 \times 10^{-3}$ V  
   (c) $9.9 \times 10^{-2}$ V  (d) $12.5 \times 10^{-4}$ V

10. Four identical loops move at the same velocity toward the right in a uniform magnetic field inside the dashed lines, as shown in Figure Q18.10. Which choice below best represents the ranking of the magnitudes (largest to smallest) of the induced currents in the loops?
   (a) $1 = 2 > 3 = 4$
   (b) $2 = 1 > 4 > 3$
   (c) $3 > 4 > 2 > 1$
   (d) $1 > 2 > 3 = 4$
   (e) $1 = 2 > 3 = 4$
(a) The secondary loop connected to the spark plug has many more turns than the primary loop attached to the battery.
(b) The current in the primary coil is reduced quickly.
(c) An iron core increases the magnetic flux through the primary and secondary coils.
(d) All three mechanisms are involved.
(e) Only mechanisms (a) and (b) are involved.

12. A respiration detector consists of a coil placed on a person’s chest and another placed on the person’s back. There is a constant current in one coil. What causes an induced current to be produced in the other coil?
(a) The person’s heart beats.
(b) The person’s breathing causes the coil separation to change.
(c) The person moves.
(d) All three of the above occur.

13. A transformer has a small number of turns in the primary coil and a large number in the secondary. The electric power input to the primary coil is \( P = I^2 R \). The secondary coil with more turns will have a greater emf (in volts) across it than the primary coil. If we connect the secondary coil across a light-bulb, we get which of the following?
(a) Extra power because of the higher emf
(b) The same power because of higher emf and lower current
(c) Less power because the current is less
(d) There is too little information to answer this question.

**Conceptual Questions**

14. A bar magnet falling with the north pole facing down passes through a coil held vertically. Sketch flux-versus-time and emf-versus-time graphs for the magnet approaching the coil and passing through it. What assumptions did you make?

15. An induction cooktop has a smooth surface. When on high, the surface does not feel warm, yet it can quickly cook soup in a metal bowl. However, it cannot cook soup in a ceramic or glass bowl. Explain how the cooktop works.

16. Use your knowledge of Lenz’s law to find the direction of the induced current in a coil when a magnet is falling through it. How many possible answers can you give?

17. Why does the magnetic field of the induced current oppose the change in the external magnetic field?

18. Describe three common applications of electromagnetic induction.

19. Two rectangular loops A and B are near each other. Loop A has a battery and a switch. Loop B has no battery. Imagine that a current starts increasing in loop A. Will there be a current in loop B? Samir argues that there will be current. Ariana argues that there will be no current. Provide experimental evidence to support the claims of both students.

20. An apnea monitor can prevent sudden infant death syndrome by sounding an alarm when a sleeping infant stops breathing. One coil carrying an alternating electric current is placed on the chest of an infant and a second coil is placed on the infant’s back. Explain how the apnea monitor detects the cessation of breathing.

21. A simple metal detector has a coil with an alternating current in it. The current produces an alternating magnetic field. If a piece of metal is near the coil, eddy currents are induced in the metal. These induced eddy currents produce induced magnetic fields that are detected by a magnetic field detection device. Draw a series of sketches representing this process, including the appropriate directions of the magnetic fields at one instant of time, and indicate two applications for this device.

22. Construct flux-versus-time and emf-versus-time graphs that explain how an electric generator works.

23. How is it possible to get a 2000-V emf from a 12-volt battery?

**Problems**

Below, \( \text{EXT} \) indicates a problem with a biological or medical focus. Problems labeled \( \text{EST} \) ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Problems marked with \( \text{F} \) require you to make a drawing or graph as part of your solution. Asterisks indicate the level of difficulty of the problem. Problems with no \( * \) are considered to be the least difficult. A single \( * \) marks moderately difficult problems. Two \( ** \) indicate more difficult problems.

### 18.1 Inducing an electric current

1. * You and your friend are performing experiments in a physics lab. Your friend claims that in general, something has to move in order to induce a current in a coil that has no battery. What experiments can you perform to support her idea? What experiments can you perform to reject it?

2. * Your friend insists that a strong magnetic field is required to induce a current in a coil that has no battery. Describe one experiment that she and you can perform to observe that a strong magnetic field helps induce an electric current and two experiments where no current is induced even with a strong magnetic field. What should you conclude about your friend’s idea?

3. You decide to use a metal ring as an indicator of induced current. If there is a current, the ring will feel warm in your hand. You place the ring around a solenoid, as shown in Figure P18.3. (a) Will the ring feel warm if there is constant nonzero current in the solenoid? (b) Will the ring feel warm if the current in the solenoid is alternating? Explain your answers.

4. * To check whether a light bulb permanently attached to a coil is still good, you place the coil next to another coil that is attached to a battery, as shown in Figure P18.4. Explain how or whether each of the following actions can help you determine if the light bulb is ok. (a) Close the switch in circuit A. (b) Keep the switch in circuit A closed. (c) Open the switch in circuit A.

5. * Flashlight without batteries A flashlight that operates without batteries is lying on your desk. The light illuminates only when you continuously squeeze the flashlight’s handle. You also notice that paper clips tend to stick to the outside

---

**Figure P18.3**

**Figure P18.4**
of the flashlight. What physical mechanism might control the operation of the flashlight?
6. You need to invent a practical application for a coil of wire that detects the vibrations or movements of a nearby magnet. Describe your invention. (The application should not repeat any described in this book.)
7. * Detect burglars entering windows. Describe how you will design a device that uses electromagnetic induction to detect a burglar opening a window in your ground floor apartment. Include drawings and a word description.
8. * A coil connected to an ammeter can detect alternating currents in other circuits. Explain how this system might work. Could you use it to eavesdrop on a telephone conversation being transmitted through a wire?

18.2 Magnetic flux
9. The \( \mathbf{B} \) field in a region has a magnitude of 0.40 T and points in the positive \( z \)-direction, as shown in Figure P18.9. Determine the magnetic flux through (a) surface abcd, (b) surface bcef, and (c) surface adef.

10. EST How do you position a bicycle tire so that the magnetic flux through it due to Earth’s magnetic field is as large as possible? Estimate this maximum flux. What assumptions did you make?

11. EST Estimate the magnetic flux through your head when the \( \mathbf{B} \) field of a 1.4-T MRI machine passes through your head.

12. EST Estimate the magnetic flux through the south- and west-facing windows of a house in British Columbia, where Earth’s \( \mathbf{B} \) field has a magnitude of \( 5.8 \times 10^{-5} \) T and points roughly north with a downward inclination of 72°. Explain how you made the estimates.

18.3 Direction of the induced current
13. * You perform experiments using an apparatus that has two insulated wires wrapped around a cardboard tube (Figure P18.4). Determine the direction of the current in the bulb when (a) the switch is closing and the current in loop A is increasing, (b) the switch has just closed and there is a steady current in loop A, and (c) the switch has just opened and the current in loop A is decreasing.

14. * You have the apparatus shown in Figure P18.14. A circular metal plate swings past the north pole of a permanent magnet. The metal consists of a series of rings of increasing radius. Indicate the direction of the current in one ring (a) as the metal swings down from the left into the magnetic field and (b) as the metal swings up toward the right out of the magnetic field. Use Lenz’s law to justify your answers.

15. * You suggest that eddy currents can stop the motion of a steel disk that vibrates while hanging from a spring. Explain how you can do this without touching the disk.

16. * Your friend thinks that an induced magnetic field is always opposite the changing external field that induces an electric current. Provide a detailed description of a situatation in which this idea would violate energy conservation.

18.4–18.7 Faraday’s law of electromagnetic induction; Skills for analyzing processes involving electromagnetic induction; Changing \( \mathbf{B} \) field magnitude and induced emf; Changing area and induced emf
17. / The magnetic flux through three different coils is changing as shown in Figure P18.17. For each situation, draw a corresponding graph showing qualitatively how the induced emf changes with time.

Figure P18.17

18. / The magnetic flux through three different coils is changing as shown in Figure P18.18. For each situation, draw a corresponding graph showing quantitatively how the induced emf changes with time.

Figure P18.18

19. A magnetic field passing through two identical coils decreases from a magnitude of \( B_0 \) to zero in the time interval \( \Delta t \). The first coil has twice the number of turns as the second.
(a) Compare the emfs induced in the coils. (b) How can you change the experiment so that the emfs produced in them are the same?

20. B10 Stimulating the brain In transcranial magnetic stimulation (TMS) an abrupt decrease in the electric current in a small coil placed on the scalp produces an abrupt decrease in the magnetic field inside the brain. Suppose the magnitude of the \( \mathbf{B} \) field changes from 0.80 T to 0 T in 0.080 s. Determine the induced emf around a small circle of brain tissue of radius \( 1.2 \times 10^{-3} \) m. The \( \mathbf{B} \) field is perpendicular to the surface area of the circle of brain tissue.

21. * To measure a magnetic field produced by an electromagnet, you use a circular coil of radius 0.30 m with 25 loops (resistance of 25 \( \Omega \)) that rests between the poles of the magnet and is connected to an ammeter. While the current in the electromagnet is reduced to zero in 1.5 s, the ammeter in the coil shows a steady reading of 180 mA. Draw a picture of the experimental setup and determine everything you can about the electromagnet.

22. You want to use the idea of electromagnetic induction to make the bulb in your small flashlight glow; it glows when the potential difference across it is 1.5 V. You have a small bar magnet and a coil with 100 turns, each with area \( 3.0 \times 10^{-4} \) m². The magnitude of the \( \mathbf{B} \) field at the front of
the bar magnet’s north pole is 0.040 T and reaches 0 T when it is about 4 cm away from the pole. Can you make the bulb light? Explain.

23. * BI0 Breathing monitor An apnea monitor for adults consists of a flexible coil that wraps around the chest (Figure P18.23). When the patient inhales, the chest expands, as does the coil. Earth’s \( \mathbf{B} \) field of \( 5.0 \times 10^{-2} \) T passes through the coil at a 53° angle relative to a line perpendicular to the coil. Determine the average induced emf in such a coil during one inhalation if the 300-turn coil area increases by 42 cm² during 2.0 s.

24. * A bar magnet induces a current in an \( N \)-turn coil as the magnet moves closer to it (Figure P18.24). The coil’s radius is \( R \) cm, and the average induced emf across the bulb during the time interval is \( \varepsilon \) V. (a) Make a list of the physical quantities that you can determine using this information; (b) Is the direction of the induced current from lead a to b, or from b to a? Explain.

25. * You have a coil of wire with 10 turns each of 1.5 cm radius. You place the plane of the coil perpendicular to a 0.40-T \( \mathbf{B} \) field produced by the poles of an electromagnet (Figure Q18.2). (a) Find the magnitude of the average induced emf in the coil when the magnet is turned off and the field decreases to 0 T in 2.4 s. (b) Is the direction of the induced current in the galvanometer from lead a to b, or from b to a? Explain.

26. * An experimental apparatus has two parallel horizontal metal rails separated by 1.0 m. A 2.0-Ω resistor is connected from the left end of one rail to the left end of the other. A metal axle with metal wheels is pulled toward the right along the rails at a speed of 20 m/s. Earth’s uniform \( 5.0 \times 10^{-5} \) T \( \mathbf{B} \) field points down at an angle of 53° below the horizontal. Make a list of the physical quantities you can determine using this information and determine two of them.

27. * Two horizontal metal rails are separated by 1.5 m and connected at their ends by a 3.0-Ω resistor to form a long, thin U shape. A metal axle with metal wheels on each side rolls along the rails at a speed of 25 m/s. Earth’s \( \mathbf{B} \) field has a magnitude of \( 5.0 \times 10^{-3} \) T and tilts downward 68° below the horizontal. Make a list of the physical quantities you can determine using this information and determine two of them.

28. A Boeing 747 with a 65-m wingspan is cruising northward at 250 m/s toward Alaska. The \( \mathbf{B} \) field at this location is \( 5.0 \times 10^{-5} \) T and points 60° below its direction of travel. Determine the potential difference between the tips of its wings.

29. A circular loop of radius 9.0 cm is placed perpendicular to a uniform 0.35-T \( \mathbf{B} \) field. You collapse the loop into a long, thin shape in 0.10 s. What is the average induced emf while the loop is being reshaped? What assumptions did you make?

30. * EST BI0 Magnetic field and brain cells Suppose a power line produces a \( 6.0 \times 10^{-4} \) T peak magnetic field 60 times each second at the location of a neuron brain cell of radius \( 6.0 \times 10^{-6} \) m. Estimate the maximum magnitude of the induced emf around the perimeter of this cell during one-half cycle of magnetic field change.

31. * You need to test Faraday’s law. You have a 12-turn rectangular coil that measures 0.20 m \( \times \) 0.40 m and an electromagnet that produces a 0.25-T magnetic field in a well-defined region that is larger than the area of the coil. You also have a stopwatch, an ammeter, a voltmeter, and a motion detector. (a) Describe an experiment you will design to test Faraday’s law. (b) How will you calculate the measurable outcome of this experiment using the materials available? (c) Describe how you can test Lenz’s law with this equipment.

32. * You build a coil of radius \( r \) m and place it in a uniform \( \mathbf{B} \) field oriented perpendicular to the coil’s surface. What is the total electric charge that passes through the coil’s wire loops if the \( \mathbf{B} \) field decreases at a constant rate to zero? The resistance of the coil’s wire is \( R \) (Ω).

33. * Equation Jeopardy 1 Invent a problem for which the following equation might be a solution.

\[
0.01 V = \frac{(100)(\cos 0°)(0.12 T - 0)}{(1.2 s - 0)}
\]

34. * Equation Jeopardy 2 Invent a problem for which the following equation might be a solution.

\[
0.01 V = \frac{100 \pi (0.10 m)^2 (0.12 T)(\cos 0° - \cos 90°)}{(t - 0)}
\]

35. * Equation Jeopardy 3 Invent a problem for which the following equation might be a solution.

\[
e = -(35) \frac{(0.12 T)(\cos 0°)(\pi (0.10 m)^2)}{(3.0 s - 0)}
\]

18.8 Changing orientation and induced emf

36. * EST Generator for space station Astronauts on a space station decide to use Earth’s magnetic field to generate electric current. Earth’s \( \mathbf{B} \) field in this region has the magnitude of \( 3.0 \times 10^{-3} \) T. They have a coil that rotates 90° in 1.2 s. The area inside the coil measures 5000 m². Estimate the number of loops needed in the coil so that during that 90° turn it produces an average induced emf of about 120 V. Indicate any assumptions you made. Is this a feasible way to produce electric energy?

37. * EST The surface of the coil of wire discussed in Problem 25 is initially oriented perpendicular to the field, as shown in Figure Q18.2. (a) Estimate the magnitude of the average induced emf if the coil is rotated 90° in 0.050 s in the 0.40-T field. The coil’s surface is now parallel to the \( \mathbf{B} \) field. (b) Determine the magnitude of the average induced emf if the coil is rotated another 90° in 0.020 s.

38. * A toy electric generator has a 20-turn circular coil with each turn of radius 1.8 cm. The coil resides in a 1.0-T \( \mathbf{B} \) field. It also has a lightbulb that lights if the potential difference across it is about 1 V. You start rotating the coil, which is initially perpendicular to the \( \mathbf{B} \) field. (a) Determine the time interval needed for a 90° rotation that will produce an average induced emf of 1.0 V. (b) Use a proportion technique to show that the same emf can be produced if the time interval for one rotation is reduced by one-fourth while the radius of the coil is reduced by one-half.

39. A generator has a 450-turn coil that is 10 cm long and 12 cm wide. The coil rotates at 8.0 rotations per second in a 0.10-T magnitude \( \mathbf{B} \) field. Determine the generator’s peak voltage.

40. * You need to make a generator for your bicycle light that will provide an alternating emf whose peak value is 4.2 V. The
generator coil has 55 turns and rotates in a 0.040-T magnitude \( \vec{B} \) field. If the coil rotates at 400 revolutions per second, what must the area of the coil be to develop this emf? Describe any problems with this design (if there are any).

41. **Evaluating a claim** A British bicycle light company advertises flashing bicycle lights that require no batteries and produce no resistance to riding. A magnet attached to a spoke on the bicycle tire moves past a generator coil on the bicycle frame, inducing an emf that causes a light to flash. The magnet and coil never touch. Does this lighting system really produce no resistance to riding? Justify your answer.

42. * The alternator in an automobile produces an emf with a maximum value of 12 V when the engine is idling at 1000 revolutions per minute (rpm). What is the maximum emf when the engine of the moving car turns at 3000 rpm?

43. * A generator has a 100-turn coil that rotates in a 0.30-T magnitude \( \vec{B} \) field at a frequency of 80 Hz (80 rotations per second) causing a peak emf of 38 V. (a) Determine the area of each loop of the coil. (b) Write an expression for the emf as a function of time (assuming the emf is zero at time zero). (c) Determine the emf at 0.0140 s.

44. * A 10-Hz generator produces a peak emf of 40 V. (a) Write an expression for the emf as a function of time. Indicate your assumptions. (b) Determine the emf at the following times: 0.025 s, 0.050 s, 0.075 s, and 0.100 s. (c) Plot these emf-versus-time data on a graph and connect the points with a smooth curve. What is the shape of the curve?

### 18.9 Transformers: Putting it all together

45. You need to build a transformer that can step the emf up from 120 V to 12,000 V to operate a neon sign for a restaurant. What will be the ratio of the secondary to primary turns of this transformer?

46. Your home’s electric doorbell operates on 10 V. Should you use a step-up or step-down transformer in order to convert the home’s 120 V to 10 V? Determine the ratio of the secondary to primary turns needed for the bell’s transformer.

47. * A 9.0-V battery and switch are connected in series across the primary coil of a transformer. The secondary coil is connected to a lightbulb that operates on 120 V. Draw the circuit. Describe in detail how you can get the bulb to light—not necessarily continuously.

48. * You are fixing a transformer for a toy truck that uses an 8.0-V emf to run it. The primary coil of the transformer is broken; the secondary coil has 30 turns. The primary coil is connected to a 120-V wall outlet. (a) How many turns should you have in the primary coil? (b) If you then connect this primary coil to a 240-V source, what emf would be across the secondary coil?

### 18.10 Mechanisms explaining electromagnetic induction

49. ** A wire loop has a radius of 10 cm. A changing external magnetic field causes an average 0.60-N/C electric field in the wire. (a) Determine the work that the electric field does in pushing 1.0 C of electric charge around the loop. (b) Determine the induced emf caused by the changing magnetic field. (c) You measure a 0.10-A electric current. What is the electrical resistance of the loop?

### General Problems

50. * Ice skater’s flashing belt You are hired to advise the coach of the Olympic ice-skating team concerning an idea for a costume for one of the skaters. They want to put a flat coil of wire on the front of the skater’s torso and connect the ends of the coil to lightbulbs on the skater’s belt. They hope that the bulbs will light when the skater spins in Earth’s magnetic field. Do you think that the system will work? If so, could you provide specifications for the device and justification for your advice?

51. **10 Hammerhead shark** A hammerhead shark (Figure P18.51) has a 0.90-m-wide head. The shark swims north at 1.8 m/s. Earth’s \( \vec{B} \) field at this location is \( 5.0 \times 10^{-5} \) T and points 30° below the direction of the shark’s travel. Determine the potential difference between the two sides of the shark’s head.

52. * Car braking system You are an inventor and want to develop a braking system that not only stops the car but also converts the original kinetic energy to some other useful energy. One of your ideas is to connect a rotor coil (the rotating coil of the generator) to the turning axle of the car. When you press on the brake pedal, a switch turns on a steady electric current to a stationary coil (an electromagnet called the stator) that produces a steady magnetic field in which the rotor turns. You now have a generator that produces an alternating current and an induced emf—electric power. Make a simple drawing of the rotor and stator at one instant and determine the direction of the magnetic force exerted on the rotor. Does this force help brake the car? Explain.

53. ** Your professor asks you to help design an electromagnetic induction sparker (a device that produces sparks). Include drawings and word descriptions for how it might work, details of its construction, and a description of possible problems.

54. ** In a new lab experiment, two parallel vertical metal rods are separated by 1.0 m. A 2.0-Ω resistor is connected from the top of one rod to the top of the other. A 0.20-kg horizontal metal bar falls between the rods and makes contact at its ends with the rods. A \( \vec{B} \) field of \( 0.50 \times 10^{-5} \) T points horizontally between the rods. The bar should eventually reach a terminal falling velocity (constant speed) when the magnetic force of the magnetic field on the induced current in the bar balances the downward force due to the gravitational pull of the Earth. (a) Develop in symbols an expression for the current through the bar as it falls. (b) Determine in symbols an expression for the magnetic force exerted on the falling bar (and determine the direction of that force). Remember that an electric current passes through it, and the bar is falling in the magnetic field. (c) Determine the final constant speed of the falling bar. (d) Is this process realistic? Explain.

55. ** EST Designing a sparker** Your friend decides to use a device that converts some mechanical energy into the production of a spark to ignite lighter fluid. Use the information and the questions below to decide whether his sparker will work. The sparker has a coil connected across a very short gap (0.1 mm) between the ends of the wire in the coil. (a) Estimate the potential difference needed across this gap to cause...
dielectric breakdown (a spark) to ignite the fumes from the wick. Dielectric breakdown occurs when the magnitude of the $E$ field is $3 \times 10^6$ V/m or greater. (b) Estimate, based on mechanical properties, the shortest time interval that you think a person can push a small magnet from several centimeters away to the surface of a coil. (c) As the magnet is pushed toward the coil, the field in the coil increases and causes an induced emf. If the magnetic flux inside one loop increases by $10^{-7}$ T $\cdot$ m$^2$ as the magnet moves forward, how many coil turns are needed to produce the emf to cause a spark? Is this a reasonable lighter system?

56. ** You have a 12-V battery, some wire, a switch, and a separate coil of wire. (a) Design a circuit that will produce an emf around the coil even though it is not connected to the battery. (b) Show, using appropriate equations, why your system will work. (c) Describe one application for your circuit.

57. * Design a burglar alarm You decide to build your own burglar alarm. Your window frames are wood, so you decide to fit the sides with metal sliders and the bottom with metal strips. These changes will turn the window area into a metal loop whose size changes as the window opens. Your idea is that an electric current will be induced in this loop as the window opens in Earth's magnetic field. The current can set off an alarm if a burglar enters. How feasible is your idea?

58. * You want to build a generator for a multi-day canoe trip. You have a fairly large permanent magnet, some wire, and a lightbulb. Design a generator and provide detailed specifications for it. (Ideas for the design could include cranking a handle or placing a paddle wheel that turns a coil in a nearby stream.)

59. ** Free energy from power line While on a camping trip, you decide to get some free electric energy. A power line is 12 m above the ground and carries an alternating current. You place a 0.50 m $\times$ 3.0 m coil with 100 turns below the wire so it lies with the 3.0-m side on the ground. The coil is connected to a light bulb. (a) Will the light bulb glow? (b) Indicate in a drawing the orientation of the coil relative to the power line so that a maximum changing flux passes through the coil. (c) If the current in the power line decreases from 200 A to 0 A in 1/240 s, what is the average emf induced in the coil? (Hint: Determine the $\vec{B}$ field produced by the long straight power line (see Chapter 17).) Describe any assumptions that you make.

60. * EST A sparker used to ignite lighter fluid in a barbeque grill is shown in Figure P18.60. You compress a knob at the end of the sparker. This compresses a spring, which when released moves a magnet at the end of the knob quickly into a 200-turn coil. The change in flux through the coil induces an emf that causes a spark across the 0.10-mm gap at the end of the sparker. (a) Estimate the time interval needed for the change in flux in order to produce this spark. Indicate any assumptions you made. (b) Is this a realistic process? Explain.

### Reading Passage Problems

#### BIO Magnetic induction tomography (MIT)

Magnetic induction tomography is an imaging method used in mineral, natural gas, oil, and groundwater exploration; as an archaeological tool; and for medical imaging. MIT has also been used to measure topsoil depth in agricultural soils. Topsoil depth is information that many farmers need: for instance, corn yield is much higher in soil that has a deep topsoil layer above the underlying, impermeable claypan. Using a trailer attached to a tractor, a farmer can map an 80,000-m$^2$ (about 20-acre) field for topsoil depth in about 1 hour.

**Figure 18.20** shows how MIT works. A time-varying electric current in a source coil (Figure 18.20a) induces a changing magnetic field that passes into the region to be imaged—in this case, the soil (Figure 18.20b). This changing magnetic field induces a weak induced electric current in topsoil and a stronger induced current in the more conductive claypan soil at the same depth. (Figure 18.20c; the current direction here is drawn as though the source current and source fields are increasing). This changing induced electric current in turn produces its own induced magnetic field (Figure 18.20d). The induced magnetic field passes out of the region being mapped to a detector coil (Figure 18.20e) near the source coil. The nature of the signal at the detector (its magnitude and phase) provides information about the region being mapped. A strong signal returned to the detector coil indicates a claypan layer near the surface; a weak signal returns if the clay layer is deeper below the surface.

### Reading Passage Problems

**BIO MRI power failure** Jose needs an MRI (magnetic resonance imaging) scan. During the exam, Jose lies in a region of a very strong 1.5-T magnetic field that points down into his chest from above. A sudden power failure causes the power supply for the magnet to shut down, reducing the magnetic field from 1.5 T to 0 T in 0.50 s. Consequently, the $\vec{B}$ field through Jose’s 0.3-m by 0.4-m chest decreases. The conductive fluid tissue inside his body along the edge of his chest is a loop, with the chest as the area inside this loop. (a) Estimate the induced emf around this conducting loop as the $\vec{B}$ field decreases. (b) If the resistance of his body tissue around this loop is 5 $\Omega$, what is the induced current passing around his body? (c) What is the direction of the current?

**Magstripe reader** A magstripe reader used to read a credit card number or a card key for a hotel room has a tiny coil that detects a changing magnetic field as tiny bar magnets pass by the coil. Calculate the magnitude of the induced emf in a magstripe card reader coil. Assume that the magstripe magnetic field changes at a constant rate of 500 mT/ms as the region between two tiny magnets on the stripe passes the coil. The reader coil is 2.0 mm in diameter and has 5000 turns.

Show that when a metal rod $L$ meters long moves at speed $v$ perpendicular to $\vec{B}$ field lines, the magnetic force exerted by the field on the electrically charged particles in the rod produces a potential difference between the ends of the rod equal to the product $BLv$.

64. ** EST The Tower of Terror ride** Figure 18.10 shows a Tower of Terror vehicle near the vertical end of its ride. (a) Is its 161-km/h speed what you would expect of an object after a 115-m fall? Explain. (b) Estimate the time interval for the free-fall part of its trip. (c) Estimate the average acceleration of the vehicle while stopping due to its magnetic braking.
69. Describe all the changes that would occur if the source current were in the direction shown in Figure 18.20 but decreasing instead of increasing.

(a) The induced current would be in the opposite direction.
(b) The induced magnetic field would be in the opposite direction.
(c) The detected current would be in the opposite direction.
(d) a and b
(e) a, b, and c

810 Measuring the motion of flying insects

Studying the motion of flying animals, particularly small insects, is difficult. One method researchers use involves attaching a tiny coil with miniature electronics to the neck of an insect and another coil to its thorax (Figure 18.21). They place the insect in a strong magnetic field and observe the changing orientations and induced emfs of the two coils in the field as the insect flies. Suppose that a 50-turn coil of radius $2.0 \times 10^{-3}$ m is attached to a tsetse fly that is flying in a $4.0 \times 10^{-3}$ T magnetic field. The tsetse fly makes a $90^\circ$ turn in $0.020$ s. Consider the average magnitude of the induced emf that occurs due to the turn of the tsetse fly and its coil.

70. Which of the quantities $B_{ex}$, $A$, or $\theta$ is changing as the fly turns?

(a) $B_{ex}$
(b) $A$
(c) $\theta$
(d) All of them
(e) None of them

71. Which answer is closest to the magnitude of the flux change?

(a) $1 \times 10^{-4}$ T·m²
(b) $2 \times 10^{-4}$ T·m²
(c) $3 \times 10^{-7}$ T·m²
(d) $5 \times 10^{-8}$ T·m²

72. Which answer is closest to the induced emf on the tsetse fly coil during the $90^\circ$ turn?

(a) $6 \times 10^{-7}$ V
(b) $1 \times 10^{-4}$ V
(c) $4 \times 10^{-9}$ V
(d) $2 \times 10^{-7}$ V

73. Which of the following could double the emf produced when the fly turns $90^\circ$?

(a) Double the number of turns in the coil.
(b) Double the coil’s area.
(c) Double the magnitude of the external magnetic field.
(d) Get the tsetse fly to take twice as long to turn.
(e) a, b, and c

Figure 18.21 The coil changes its orientation with respect to the external magnetic field as the fly makes a turn.