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The path followed by a bottlenose dolphin jumping out of the water is called a parabola. A parabola is a visual realisation of a quadratic function of the form \( y = -x^2 + k \). Using this, scientists can calculate the height of a dolphin’s jump (in the \( y \)-axis) and the distance travelled (in the \( x \)-axis).

There is no scientific agreement about why dolphins jump. Some scientists believe it is because they are trying to conserve energy, some believe it is to help them find food, and others believe they do it just for fun.

The parabola is a beautiful and elegant shape, commonly seen in nature. It is also seen in many man-made structures such as bridges and other buildings.

**LEARNING OBJECTIVES**

- Factorise quadratic expressions where the coefficient of \( x^2 \) is greater than 1
- Complete the square and use this to solve quadratic equations
- Solve quadratic equations using the quadratic formula
- Understand and use the discriminant to identify whether the roots are (i) equal and real, (ii) unequal and real or (iii) not real
- Understand the roots \( \alpha \) and \( \beta \) and how to use them

**BASIC PRINCIPLES**

1. Factorise
   - \( a \) \( 6x^2 + 9x \)
   - \( b \) \( 2b^2 + 8b \)
   - \( c \) \( 9m^2 - 27m \)
   - \( d \) \( 9xy^2 + 36x^2y \)
   - \( e \) \( 24x - 64x^2 \)

2. Factorise
   - \( a \) \( x^2 + 9x + 18 \)
   - \( b \) \( x^2 - 7x + 12 \)
   - \( c \) \( x^2 - 2x - 3 \)
   - \( d \) \( x^2 + 15x + 36 \)
   - \( e \) \( x^2 + 12x + 27 \)

3. Factorise
   - \( a \) \( x^2 - 9 \)
   - \( b \) \( x^2 - 25 \)
   - \( c \) \( 9x^2 - 16 \)
   - \( d \) \( 25x^2 - 16 \)

**HINT**

These questions are all examples of the difference of two squares.
2.1 Factorise Quadratic Expressions Where the Coefficient of \( x^2 \) Is Greater Than 1

**SKILLS: DECISION MAKING, CRITICAL THINKING**

Factorise \( 3x^2 + 5x - 2 \)

We need to find two numbers:
They need to **add** together to make +5 (the coefficient of \( x \)), and they need to **multiply** together to give -6 (the coefficient of \( x^2 \times \) the constant term, in this case \( 3 \times -2 \))
\[
6 \times -1 = -6 \quad \text{and} \quad 6 + (-1) = 5
\]
These numbers are then used to split the \( 5x \) into two terms, \( 6x \) and \( -1x \)
\[
3x^2 + 5x - 2 = 3x^2 + 6x - x - 2 = 3x(x + 2) - (x + 2) = (x + 2)(3x - 1)
\]

**EXAMPLE 1**

The \( 5x \) term has been split into \( 6x \) and \( -x \)
The \( 3x^2 + 6x \) term and the \( -x - 2 \) term have both been factorised

**SKILLS: CRITICAL THINKING, DECISION MAKING**

Factorise \( 6x^2 - 5x - 4 \)

Find two numbers that add together to make -5 and multiply together to give -24
(the coefficient of \( x^2 \) multiplied by the constant term, in this case \( 6 \times -4 \))
\[
3 \times -8 = -24 \quad \text{and} \quad 3 + -8 = -5
\]
\[
6x^2 - 5x - 4 = 6x^2 + 3x - 8x - 4 = 3x(2x + 1) - 4(2x + 1) = (2x + 1)(3x - 4)
\]

**EXAMPLE 2**
The \( -5x \) term has been split into \( 3x \) and \( -8x \)
The \( 6x^2 + 3x \) term and the \( -8x - 4 \) term have both been factorised

**EXERCISE 1**

**SKILLS: CRITICAL THINKING, DECISION MAKING**

1. Factorise
   \[
   \begin{align*}
   a) & \quad 3x^2 - 7x - 6 \\
   b) & \quad 2x^2 + 11x + 5 \\
   c) & \quad 2x^2 - 7x - 4 \\
   d) & \quad 5x^2 - 16x + 3 \\
   e) & \quad 6x^2 + x - 12 \\
   f) & \quad 9x^2 - 6x + 1 \\
   g) & \quad 9x^2 - 18x - 7 \\
   \end{align*}
   \]

2. Solve
   \[
   \begin{align*}
   a) & \quad 2x^2 + 5x + 2 = 0 \\
   b) & \quad 4x^2 + 17x + 4 = 0 \\
   c) & \quad 3x^2 - 13x = -4 \\
   d) & \quad 6x^2 - 10x + 4 = 0 \\
   e) & \quad 2(2x^2 + 3) = -15x \\
   f) & \quad 3(x^2 - 2) = -17x \\
   g) & \quad 15x^2 + 42x - 9 = 0 \\
   h) & \quad 3x(3x - 4) = -4 \\
   \end{align*}
   \]
2.2 COMPLETE THE SQUARE AND USE THIS TO SOLVE QUADRATIC EQUATIONS

A perfect square quadratic is in the form:

\[ x^2 + 2bx + b^2 = (x + b)^2 \quad \text{or} \quad x^2 - 2bx + b^2 = (x - b)^2 \]

To turn a non-perfect square into a perfect square you need to manipulate the expression.

To complete the square of the function \( x^2 + 12x \) you need a further term \( b^2 \).

So the completed square form is:

\[ x^2 + 2bx = (x + b)^2 - b^2 \]

Similarly

\[ x^2 - 2bx = (x - b)^2 - b^2 \]

Completing the square:

\[ x^2 + 12x = \left( x + \frac{b}{2} \right)^2 - \left( \frac{b}{2} \right)^2 \]

**SKILLS: REASONING, EXECUTIVE FUNCTION**

Complete the square for

\[ a \ x^2 + 12x \quad \quad b \ 2x^2 - 10x \]

\[ a \ x^2 + 12x \]

\[ = (x + 6)^2 - 36 \]

\[ = (x + 6)^2 - 6^2 \]

\[ 2b = 12 \text{ so } b = 6 \]

\[ b \ 2x^2 - 10x \]

Here the coefficient of \( x^2 \) is 2

So take out the coefficient of \( x^2 \)

Complete the square on \( (x^2 - 5x) \)

\[ = 2 \left[ \left( x - \frac{5}{2} \right)^2 - \left( \frac{5}{2} \right)^2 \right] \]

\[ = 2 \left( x - \frac{5}{2} \right)^2 - 25 \]

By completing the square we can solve any quadratic equation.

**EXAMPLE 3**

Complete the square to solve

\[ a \ x^2 + 8x + 10 = 0 \quad \quad b \ 2x^2 - 8x + 7 = 0 \]

\[ a \ x^2 + 8x + 10 = 0 \]

Check coefficient of \( x^2 = 1 \)

Subtract 10 to get LHS in the form \( ax^2 + bx \)

Complete the square for \( (x^2 + 8x) \)

Add \( 4^2 \) to both sides

Square root both sides

Subtract 4 from both sides

Then the solutions of \( x^2 + 8x + 10 = 0 \) are either

\[ x = -4 \pm \sqrt{6} \]

\[ x = -4 + \sqrt{6} \text{ or } x = -4 - \sqrt{6} \]

Leave your answer in surd form
b) \[2x^2 - 8x + 7 = 0\]
\[x^2 - 4x + \frac{7}{2} = 0\]
\[x^2 - 4x = -\frac{7}{2}\]
\[(x - 2)^2 - (2)^2 = -\frac{7}{2}\]
\[(x - 2)^2 = \frac{1}{2}\]
\[x - 2 = \pm \frac{1}{\sqrt{2}}\]
\[x = 2 \pm \frac{1}{\sqrt{2}}\]

So the roots are either
\[x = 2 + \frac{1}{\sqrt{2}}\] or \[x = 2 - \frac{1}{\sqrt{2}}\]

**EXERCISE 2**

**SKILLS: PROBLEM SOLVING**

1. Complete the square for these expressions.
   - a) \[x^2 + 4x\]
   - b) \[x^2 - 16x\]
   - c) \[3x^2 - 24x\]
   - d) \[x^2 - x - 12\]
   - e) \[x^2 + x - 1\]
   - f) \[3x^2 - 6x + 1\]
   - g) \[2x^2 + 3x - 1\]
   - h) \[4x^2 + 6x - 1\]

2. Solve these quadratic equations by completing the square.
   Leave your answer in surd form where appropriate.
   - a) \[6x^2 - 11x - 10 = 0\]
   - b) \[x^2 + 2x - 2 = 0\]
   - c) \[2x^2 - 6x + 1 = 0\]
   - d) \[2x^2 + 3x - 6 = 0\]
   - e) \[4x^2 - 59x - 15 = 0\]
   - f) \[4x^2 + 8x - 9 = 0\]
   - g) \[15 - 6x - 2x^2 = 0\]
   - h) \[4x^2 - x - 8 = 0\]

3. Show by completing the square that the solutions to \[ax^2 + bx + c = 0\] are \[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

**2.3 SOLVE QUADRATIC EQUATIONS USING THE QUADRATIC FORMULA**

The quadratic formula can be used to solve any quadratic equation of the form \[ax^2 + bx + c = 0\]

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

**SKILLS: PROBLEM SOLVING**

Use the quadratic formula to solve \[4x^2 - 3x - 2 = 0\]

\[x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-2)}}{2 \times 4}\]
\[x = \frac{3 \pm \sqrt{9 + 32}}{8}\]
\[x = \frac{3 \pm \sqrt{41}}{8}\]

Then \[x = \frac{3 + \sqrt{41}}{8}\] or \[x = \frac{3 - \sqrt{41}}{8}\]

Leave your answer in surd form.
SKILLS: EXECUTIVE FUNCTION

1 ▶ Solve these equations using the quadratic formula. Leave your answer in surd form where appropriate.

- **a** \(x^2 = 6x\)
- **b** \(2x^2 + 50x = 0\)
- **c** \(x^2 - x - 6 = 0\)
- **d** \(p^2 - 4p + 2 = 0\)
- **e** \(m^2 + 2m - 2 = 0\)
- **f** \(x^2 + 7x + 10 = 0\)
- **g** \(t^2 - 5t - 6 = 0\)
- **h** \(x^2 + 2x - 35 = 0\)
- **i** \(n^2 - 4n + 4 = 0\)
- **j** \(x^2 + 6x + 6 = 0\)

2 ▶ Solve these equations, leaving your answer in surd form:

- **a** \(9x^2 - 6x + 25 = 0\)
- **b** \(3x^2 - 6x + 2 = 0\)
- **c** \(6x^2 - 5x - 6 = 0\)
- **d** \(2x^2 + 3x - 1 = 0\)
- **e** \(2x^2 + 7x + 3 = 0\)
- **f** \(3x^2 + 7x + 1 = 0\)
- **g** \(4x^2 - 16x + 15 = 0\)
- **h** \(7x^2 + 5x - 3 = 0\)
- **i** \(10x^2 - 15x - 8 = 0\)
- **j** \(6x^2 + 3x - 0 = 0\)
- **k** \((x - 7)^2 = 36\)
- **l** \(4x^2 + 15x + 13 = 0\)

3 ▶ The diagram shows the floor plan of a bedroom. The total area is 35.5m. Find the value of \(m\).

2.4 UNDERSTAND AND USE THE DISCRIMINANT TO IDENTIFY WHETHER THE ROOTS ARE (i) EQUAL AND REAL, (ii) UNEQUAL AND REAL OR (iii) NOT REAL

The equation \(ax^2 + bx + c = 0\) has two solutions:

\[x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}\]

These two solutions may be the same or different, real numbers or not real numbers.

The nature of the roots (solutions) of the equation will clearly depend on the expression \(b^2 - 4ac\) under the square root sign. This expression \(b^2 - 4ac\) is called the discriminant, as it allows us to identify (discriminate) whether the roots of a particular equation are equal and real, unequal and real or not real at all.

If:
- \(b^2 - 4ac > 0\) the roots of the equation are unequal and real (two roots)
- \(b^2 - 4ac = 0\) the roots of the equation are equal and real (one root)
- \(b^2 - 4ac < 0\) there are no real roots of the equation (no roots)

EXAMPLE 6

Say what can you deduce from the values of the discriminants of these equations, and find the roots where possible.

- **a** \(2x^2 - 3x + 5 = 0\)
- **b** \(3x^2 - x - 1 = 0\)
- **c** \(4x^2 - 12x + 9 = 0\)
### Example 7

The equation $kx^2 - 2x - 8 = 0$ has two real roots.

What can you deduce about the value of the constant $k$?

Since the equation has two real roots, we know that the discriminant $b^2 - 4ac$ must be greater than zero.

We substitute $a = k$, $b = (-2)$ and $c = (-8)$ into the inequality $b^2 - 4ac > 0$, giving

\[
(-2)^2 - 4 \times k \times -8 > 0
\]

\[
4 + 32k > 0
\]

\[
k > \frac{-4}{32} \quad k > \frac{1}{8}
\]

### Exercise 4

**Skills: Reasoning**

1. Use the discriminant to determine whether these equations have one root, two roots or no roots.

   a. $x^2 - 2x + 1 = 0$
   b. $x^2 - 3x - 2 = 0$
   c. $2x^2 - 3x - 4 = 0$
   d. $2x^2 - 4x + 5 = 0$
   e. $2x^2 - 4x + 2 = 0$
   f. $2x^2 - 7x + 3 = 0$
   g. $3x^2 - 6x + 5 = 0$
   h. $7x^2 - 14x + 57 = 0$
   i. $16x^2 - 2x + 3 = 0$
   j. $x^2 + 22x + 121 = 0$
   k. $5x^2 - 4x + 81 = 0$
2. The equation \( px^2 - 2x - 7 = 0 \) has two real roots. What can you deduce about the value of \( p \)?

3. The equation \( 3x^2 + 2x + m = 0 \) has equal roots. Find the value of \( m \).

### 2.5 UNDERSTAND THE ROOTS \( \alpha \) AND \( \beta \) AND HOW TO USE THEM

If \( \alpha \) and \( \beta \) are the roots of the equation \( ax^2 + bx + c = 0 \) then we deduce that
\[
(x - \alpha)(x - \beta) = 0
\]
We can rewrite this as \( x^2 - x(\alpha + \beta) + \alpha\beta = 0 \)
Comparing this with \( ax^2 + bx + c = 0 \) we can see that
\[
\alpha + \beta = \frac{-b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}
\]

**KEY POINTS**

For the equation \( ax^2 + bx + c = 0 \)
- The sum of roots, \( \alpha + \beta = \frac{-b}{a} \)
- The product of the roots, \( \alpha\beta = \frac{c}{a} \)

### EXAMPLE 8

#### SKILLS: REASONING, CRITICAL THINKING

1. The roots of the equation \( 3x^2 + x - 6 = 0 \) are \( \alpha \) and \( \beta \)
   - Find an expression for \( \alpha + \beta \) and an expression for \( \alpha\beta \)
   - Hence find an expression for \( \alpha^2 + \beta^2 \) and an expression for \( \alpha^2\beta^2 \)
   - Find a quadratic equation with roots \( \alpha^2 \) and \( \beta^2 \)

   \[
   \begin{align*}
   a & \quad 3x^2 + x - 6 = 0 \\
   b & \quad x^2 + \frac{1}{3}x - 2 = 0 \\
   \text{Therefore sum of the roots} & \quad \alpha + \beta = \frac{-1}{3}
   \end{align*}
   \]

   **Note:** Sometimes you will need to manipulate the expressions to help you solve the questions.

   - Product of the roots \( \alpha\beta = -2 \)
   - \( \alpha + \beta^2 = \alpha^2 + 2\alpha\beta + \beta^2 \)
     
     Therefore \( \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \)
     
     Substituting the results from part a, we find
     \[
     \begin{align*}
     \alpha^2 + \beta^2 & = \left(\frac{-1}{3}\right)^2 - 2(-2) \\
     & = \frac{41}{9} \quad \text{or} \quad \frac{37}{9} \\
     \alpha^2\beta^2 & = (\alpha\beta)^2 = (-2)^2 = 4
     \end{align*}
     \]

   Divide the equation by 3 to obtain an equation where the coefficient of \( x^2 \) is 1.

   **The sum of roots:** \( \alpha + \beta = \frac{-b}{a} \)

   **The product of the roots:** \( \alpha\beta = \frac{c}{a} \)
Let the equation be \( x^2 + px + q = 0 \)

\[
p = (\alpha^2 + \beta^2) = -\frac{37}{9}
\]

\[
q = \alpha^2 \beta^2 = 4
\]

So the equation is: \( x^2 - \frac{37}{9}x + 4 = 0 \)

OR \( 9x^2 - 37x + 36 = 0 \)

**Note:** This question can be answered without finding \( \alpha \) and \( \beta \). Sometimes \( \alpha \) and \( \beta \) are not even real numbers.

The roots of the equation \( x^2 - 3x - 2 = 0 \) are \( \alpha \) and \( \beta \).

Without finding the value of \( \alpha \) and \( \beta \), find the equations with the roots

\( a \) \( 3\alpha, 3\beta \) \hspace{1cm} \( b \) \( \frac{1}{\alpha}, \frac{1}{\beta} \) \hspace{1cm} \( c \) \( \alpha^2, \beta^2 \)

If \( \alpha \) and \( \beta \) are the roots of \( x^2 - 3x - 2 = 0 \) then \( \alpha + \beta \) and \( \alpha \beta = -2 \)

\( a \) If the roots are \( 3\alpha \) and \( 3\beta \) then

Sum of the roots \( 3\alpha + 3\beta = 3 \times 3 = 9 \)

Product of the roots \( 3\alpha \times 3\beta = 9\alpha\beta = -18 \)

Equation is: \( x^2 - 9x - 18 = 0 \)

\( b \) If the roots are \( \frac{1}{\alpha}, \frac{1}{\beta} \)

Sum of roots \( \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{3}{-2} = -1.5 \)

Product of roots \( \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{-2} = 0.5 \)

Equation is: \( x^2 + \frac{3}{2}x - \frac{1}{2} = 0 \)

i.e. \( 2x^2 + 3x - 1 = 0 \)

\( c \) If the roots are \( \alpha^2, \beta^2 \) then

Sum of roots \( \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 3^2 - 2(-2) = 13 \)

Product of roots \( \alpha^2\beta^2 = \alpha\beta^2 = (-2)^2 = 4 \)

Therefore equation is \( x^2 - 13x + 4 = 0 \)
SKILLS: REASONING, CRITICAL THINKING

1. The roots of the equation \(x^2 + 5x + 2 = 0\) are \(\alpha\) and \(\beta\).
   Find an equation whose roots are
   a. \(2\alpha + 1\) and \(2\beta + 1\)
   b. \(\alpha\beta\) and \(\alpha^2\beta^2\)

2. The roots of the equation \(x^2 + 6x + 1 = 0\) are \(\alpha\) and \(\beta\).
   Find an equation whose roots are
   a. \(\alpha + 3\) and \(\beta + 3\)
   b. \(\frac{\alpha}{\beta}\) and \(\frac{\beta}{\alpha}\)

3. The roots of the equation \(x^2 - x - 1 = 0\) are \(\alpha\) and \(\beta\).
   Find an equation whose roots are
   a. \(\frac{1}{\alpha}\) and \(\frac{1}{\beta}\)
   b. \(\frac{\alpha}{\alpha + \beta}\) and \(\frac{\beta}{\alpha + \beta}\)
EXAM PRACTICE: CHAPTER 2

1. The equation \( x^2 + (p - 3)x + (3 - 2p) = 0 \), where \( p \) is a constant, has two distinct real roots.
   a. Show that \( p \) satisfies \( p^2 + 2p - 3 > 0 \) \([1]\)
   b. Find the possible values of \( p \). \([2]\)

2. a. Show that \( x^2 + 6x + 11 \) can be written as \((x + a)^2 + b\) \([2]\)
   b. Find the value of the discriminant. \([2]\)

3. Factorise completely
   a. \( 5x^2 + 16x + 3 \) \([3]\)
   b. \( 3x^2 - 7x + 4 \) \([3]\)

4. Solve these equations by completing the square:
   a. \( p^2 + 3p + 2 = 0 \) \([3]\)
   b. \( 3x^2 + 13x - 10 = 0 \) \([3]\)

5. Solve these equations by using the quadratic formula:
   a. \( 5x^2 + 3x - 1 = 0 \) \([2]\)
   b. \( (2x - 5)^2 = 7 \) \([3]\)

6. \( 4x - 5 - x^2 = b - (x + a)^2 \) where \( a \) and \( b \) are integers.
   a. Find the value of \( a \) and \( b \). \([2]\)
   b. Calculate the discriminant. \([2]\)

7. Solve \( \frac{4}{2x + 1} - 3 = -\frac{1}{4x^2 - 1} \) \([3]\)

8. Given that the roots of the equation \( 2x^2 - 7x + 3 = 0 \) are \( \alpha \) and \( \beta \), find the exact value of
   a. \( \alpha^2 + \beta^2 \) \([2]\)
   b. \( \alpha - \beta \) \([3]\)
   c. \( \alpha^3 - \beta^3 \) \([3]\)

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1. $x^2 - y^2 = (x - y)(x + y)$ is known as the *difference of two squares*. It is important to spot this in exams.

2. Quadratic equations can be solved by
   - **a** factorisation.
   - **b** completing the square: $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
   - **c** using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3. The *discriminant* of a quadratic expression is $b^2 - 4ac$

4. If $\alpha$ and $\beta$ are the roots of the equation $ax^2 + bx + c = 0$ then
   - **i** $\alpha + \beta = -\frac{b}{a}$
   - **ii** $\alpha\beta = \frac{c}{a}$
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