Teaching for Conceptual Understanding: Ratios and Proportional Relationships

Professional Development
PARTICIPANT WORKBOOK
Sampler
<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching for Conceptual Understanding:</td>
<td></td>
</tr>
<tr>
<td>Ratios and Proportional Relationships Overview</td>
<td>4</td>
</tr>
<tr>
<td>Section 1: The Progression of Ratios and Proportional Relationships</td>
<td>7</td>
</tr>
<tr>
<td>Section 2: Building the Foundation for</td>
<td></td>
</tr>
<tr>
<td>Ratios and Proportional Relationships</td>
<td>9</td>
</tr>
<tr>
<td>Section 3: Making Connections within Mathematics</td>
<td>31</td>
</tr>
<tr>
<td>Section 4: Planning Lessons with Ratios and Proportional Relationships</td>
<td>35</td>
</tr>
<tr>
<td>Reflection and Closing</td>
<td>40</td>
</tr>
<tr>
<td>References</td>
<td>41</td>
</tr>
</tbody>
</table>
Teaching for Conceptual Understanding: Ratios and Proportional Relationships Overview

Agenda

<table>
<thead>
<tr>
<th>Introduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section 1: The Progression of Ratios and Proportional Relationships</td>
</tr>
<tr>
<td>Section 2: Building the Foundation for Ratios and Proportional Relationships</td>
</tr>
<tr>
<td>Section 3: Making Connections within Mathematics</td>
</tr>
<tr>
<td>Section 4: Planning Lessons with Ratios and Proportional Relationships</td>
</tr>
<tr>
<td>Reflection and Closing</td>
</tr>
</tbody>
</table>

Outcomes

At the conclusion of this workshop, you will be able to

- articulate the learning progressions necessary for students to conceptually understand ratios and proportional relationships;
- identify strategies for helping students build their mathematical understanding of ratios and proportional relationships;
- use a planning template to build lessons that strategically support the conceptual development of ratios and proportional relationships;
- identify strategies that support simultaneous development of conceptual understanding and problem-solving skills with the intentional use of purposeful student struggle, flexible grouping, and ongoing assessments; and
- articulate common misconceptions as opportunities for students’ conceptual understanding of ratios and proportional relationships.
Content Warm-up

Problem 1: Tammy bought 3 widgets for $2.40. At the same price, how much would 10 widgets cost?

Problem 2: Tammy bought 4 widgets for $3.75. How much would a dozen widgets cost?
Section 1: The Progression of Ratios and Proportional Relationships

Exploring the Standards

**RED:**


**YELLOW:**


**GREEN:**


*Impact Observations*


Teaching for Conceptual Understanding: Ratios and Proportional Relationships
© 2013 Pearson, Inc.
The Importance of Ratios and Proportional Reasoning

1. _______________________________________________________________
   _______________________________________________________________

2. _______________________________________________________________
   _______________________________________________________________

3. _______________________________________________________________
   _______________________________________________________________

4. _______________________________________________________________
   _______________________________________________________________

5. _______________________________________________________________
   _______________________________________________________________

Revisit Section 1 Big Questions

• How are you responsible for teaching new content at your grade level?
  _______________________________________________________________
  _______________________________________________________________
  _______________________________________________________________
  _______________________________________________________________
  _______________________________________________________________

• How does what you teach impact student understanding at the next grade level?
  _______________________________________________________________
  _______________________________________________________________
  _______________________________________________________________
  _______________________________________________________________
Reflection: The Journey Begins

Write a short paragraph or story that describes the journey that your students will take during their formal study of ratios and proportional relationships.
Defining Ratio

What is a ratio?

Create at least three scenarios that the ratio 3:1 can represent.
Section 2: Building the Foundation for Ratios and Proportional Relationships

Vocabulary and Symbols in 6.RP.1

Sorting Representations

1. Use the Four Middle Schools activity sheet (page 19) and six blank index cards. The sheet contains 24 cards describing four middle schools. Each card contains information about one of the schools.

For each school, find a set of six matching cards. In each set there should be:

- A data card
- A card that gives the ratio in colon form
- A decimal card
- A fraction card
- A percent card
- A card that describes the ratio in words

Make notes about how you decide to sort the cards, and record any computations that you use.

On one of the blank index cards, write the number of boys and girls in your school. Then, make a card that uses colon form, a decimal card, a fraction card, a percent card, and a card that describes the ratio in words.

Example

<table>
<thead>
<tr>
<th>School A</th>
<th>School B</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are 420 boys and 420 girls.</td>
<td>There are 360 boys and 720 girls.</td>
</tr>
<tr>
<td>The ratio of boys to girls is 1:1.</td>
<td>The ratio of girls to boys is 1.25.</td>
</tr>
<tr>
<td>There are 420 boys and 420 girls.</td>
<td></td>
</tr>
<tr>
<td>The ratio of boys to girls is 4:5.</td>
<td></td>
</tr>
<tr>
<td>Two out of every three students are girls.</td>
<td></td>
</tr>
<tr>
<td>54% of the students are girls.</td>
<td></td>
</tr>
<tr>
<td>The ratio of girls to all students is 2:5.</td>
<td></td>
</tr>
<tr>
<td>School D</td>
<td>School C</td>
</tr>
<tr>
<td>There are 880 boys and 280 girls.</td>
<td>There are 360 boys and 450 girls.</td>
</tr>
<tr>
<td>One out of every two students is a girl.</td>
<td>There are five girls for every twenty-two boys.</td>
</tr>
<tr>
<td>There are five girls for every twenty-two boys.</td>
<td></td>
</tr>
<tr>
<td>Approximately 61% of all students are boys.</td>
<td></td>
</tr>
<tr>
<td>2/5 of the students are girls.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>


(America’s Choice 2009b, 9, 10, B1)
3. For which school was it most difficult to find the matching set of ratio, fraction, decimal, and percent cards? Say why.

4. For which school was it the easiest? Say why.

5. Think about the number of students in one of your math classes. How many students are in the class? How many of the students are boys?
   a. Use this context to give an example of a part-part comparison.
   b. Use this context to give two examples of part-whole comparisons.
   c. One way to describe this situation is to say that (number) out of (second number) of the students are boys. Write two more ways that describe the ratio of boys to all students.

(America’s Choice 2009b, 9, 10, B1)
<table>
<thead>
<tr>
<th>School A</th>
<th>School B</th>
<th>School C</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are 420 boys and 420 girls.</td>
<td>The ratio of girls to boys is 1:25.</td>
<td>There are 360 boys and 450 girls.</td>
</tr>
<tr>
<td>The ratio of boys to girls is 4:5.</td>
<td>Two out of every three students are girls.</td>
<td>The ratio of girls to all students is approximately 0.8.</td>
</tr>
<tr>
<td>Girls are approximately 67% of all students.</td>
<td>The ratio of girls to all students is 2:3.</td>
<td>The ratio of boys to all students is 0.5.</td>
</tr>
<tr>
<td>The ratio of girls to all students is 5:27.</td>
<td>( \frac{5}{9} ) of the students are girls.</td>
<td>( \frac{22}{27} ) of all students are boys.</td>
</tr>
<tr>
<td>One out of every two students is a girl.</td>
<td>There are five girls for every twenty-two boys.</td>
<td></td>
</tr>
<tr>
<td>Approximately 81% of all students are boys.</td>
<td>The ratio of boys to all students is approximately 0.8.</td>
<td></td>
</tr>
</tbody>
</table>

(America's Choice 2009b, 9, 10, B1)
### Vocabulary and Symbols in 6.RP.1

<table>
<thead>
<tr>
<th>Word Wall Artifacts Vocabulary</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Misconceptions

List misconceptions that students might have about ratios that would prevent them from succeeding with this card sort.

- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
-
Additive versus Multiplicative Reasoning

Could 7 be the missing number in this proportion? 3 : 4 :: 6 : ___ (Pearson Achievement Solutions 2007, 3)

Is it additive or multiplicative?

1. Janet and Jeanette were walking to school, each walking at the same rate. Jeanette started first. When Jeanette has walked 6 blocks, Janet has walked 2 blocks. How far will Janet be when Jeanette is at 12 blocks?

2. Lisa and Linda are planting corn on the same farm. Linda plants 6 rows. If Linda’s corn is ready to pick in 8 weeks, how many weeks will it take for Lisa’s corn to be ready?

3. Kendra and Kevin are baking cookies using the same recipe. Kendra makes 6 dozen and Kevin makes 3 dozen. If Kevin is using 6 ounces of chocolate chips, how many ounces will Kendra need?

(Van de Walle, Karp, and Bay-Williams 2013, 359)
Models and Strategies for Problem Solving in 6.RP.3

There are several different strategies listed in the standard that students can use to solve problems that involve ratios. When students first start using these strategies, their approaches may be very in-depth and lengthy. As they become more familiar with the relationships and comparisons among equivalent ratios, their strategies will become more abbreviated and efficient.

(The Common Core Standards Writing Team 2011, 6)

Tables of Equivalent Ratios

Use a ratio table to solve the following problem:

Tyler and Isaac have both volunteered to bring snacks for the class field trip. Tyler’s snack recipe calls for 1 cup of chocolate chips for every 3 cups of nuts, and Isaac’s snack recipe calls for 3 cups of chocolate chips for every 5 cups of nuts.

1. When both students’ batches of snacks have the same amount of chocolate chips, whose batch is “nuttier”?

2. When both students’ batches of snacks have the same amount of nuts, whose is more “chocolaty”?

3. If both Tyler and Isaac bring in the same number of cups of their snack, whose is “nuttier” and whose is more “chocolaty”?
Consider the following ratio:

“For every 5 cups grape juice, mix in 2 cups peach juice.”

(The Common Core Standards Writing Team 2011, 5)

Now, read through the following CCSSM standards, and consider the connections between these standards as seen in the equivalent ratio tables and graphs below:

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

(NGA Center and CCSSO 2010, 42)
Double Number Line Diagrams

Double number line diagrams used for situations with different units

Tape Diagrams

Representing a problem with a tape diagram
Slimy Gloopy mixture is made by mixing glue and liquid laundry starch in a ratio of 3 to 2. How much glue and how much starch is needed to make 85 cups of Slimy Gloopy mixture?

<table>
<thead>
<tr>
<th>Glue:</th>
<th>5 parts</th>
<th>85 cups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starch:</td>
<td>1 part</td>
<td>$85 \div 5 = 17$ cups</td>
</tr>
<tr>
<td></td>
<td>3 parts</td>
<td>$3 \cdot 17 = 51$ cups</td>
</tr>
<tr>
<td></td>
<td>2 parts</td>
<td>$2 \cdot 17 = 34$ cups</td>
</tr>
</tbody>
</table>

51 cups glue and 34 cups starch are needed.

Tape diagrams can be useful aids for solving problems.

(The Common Core Standards Writing Team 2011, 7)
Solving Problems Using 6.RP.3 Models and Strategies

**Problem A.**
Ricky Racer can complete $\frac{3}{4}$ of a race in 24 minutes. If he continues at that same rate, how long would it take him to complete the whole race? If he wanted to cut his time by 25%, how long would it take him to complete $\frac{1}{2}$ of the race? (tape diagram or double number line)

**Problem B.**
If two bags of candy cost $5, how much will 12 bags of candy cost? How much will 3 bags of candy cost? What about 6 bags of candy? (table and graph or equation)
Problem C.
At the farmers’ market, Jones sells tomatoes at 5 pounds for $7 whereas Smith sells his tomatoes at 4 pounds for $6. Which farmer sells his tomatoes at a cheaper price? (ratio table)

Problem D.
To create her special paint color for the school project, Ms. Smith combines cups of red, blue, and white paint in a ratio of 2:2:4. How much of each paint will need to be mixed to create 48 gallons of paint? (tape diagram)
Percent as a Rate per Hundred in 6.RP.3c

Solve each problem using a double number line. When you finish all four problems, go back and solve each one using a different method. Try to use a different approach for your second method for each problem.

Ann and Carmen each went on bicycle trips. Ann’s whole trip was 150 miles long. Carmen’s whole trip was 300 miles long. Each girl rides 30 miles each day.

1. What percent of her total trip does each girl complete in a day?
2. After how many days will each girl complete 40% of her trip?
3. At the start, each girl had 12 pounds of food. Unfortunately, each girl used up 75% of that food in the first 25% of the trip. When this happened:
   a. How many miles had each girl ridden?
   b. How many pounds of food did each girl have left?
4. 80% of the way through the trip, each girl will be joined by a friend. For how many miles will each girl ride with a friend?

(America’s Choice 2002, 15)
Grade 6 Progression Reflection

For Grade 6 teachers, how will you prepare to teach the CCSSM Grade 6 Ratios and Proportional Relationships concepts?

For Grade 7 teachers, what will you expect your incoming Grade 7 students to know about ratios and proportions?
Ratios with Rational Numbers in 7.RP.1
To make Super Sour Candies you mix 1/2 cup lemon juice for every 1/3 cup sugar. If you want to make 20 total cups of candy mix, how many cups of lemon juice and sugar will you need?

Defining Proportional Relationships
What is a proportional relationship?
Recognizing Proportional Relationships in 7.RP.2a

Example #1

This is an example because . . .

Example #2

This is an example because . . .

Nonexample #1

This is a nonexample because . . .

Nonexample #2

This is a nonexample because . . .
Graphing Proportional Relationships

1. Two of these four tables of values are ratio tables and two are not.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>1.2</td>
<td>1.0</td>
<td>8</td>
<td>4.4</td>
</tr>
<tr>
<td>y</td>
<td>1</td>
<td>0.5</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>1.0</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>1.5</td>
<td>3</td>
<td>15.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.0</td>
<td>4</td>
<td>19.8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.5</td>
<td>5</td>
<td>26.4</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>2.5</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

a. Which two tables are ratio tables? (This means that they represent a proportional relationship.)

b. Write a formula in the form of $y = kx$ for the two proportional relationships.

c. Use your formulas to find the missing values in the tables that represent proportional relationships.
2. Sketch and label graphs that represent the four relationships in problem 1.
3. Say how you can determine whether two quantities are proportional to each other. Use your results from problems 1 and 2 to support your answer.

4. A relationship can have two variables and still not be proportional. Say why.

5. Look again at the graphs you sketched in problem 2. What do you observe when you compare the graphs that represent proportional relationships to the graphs that represent non-proportional relationships?

6. In proportional relationships, when one variable equals zero, the other variable must also equal zero. Say why, using what you know about the tables, formulas, and graphs of proportional relationships to support your answer.

7. Sometimes, a proportional relationship has meaning for only some values. Say why, and give an example to support your reasoning.

(America’s Choice 2009a, 98, B5)
## Conceptual Understanding of Multiple Representations

<table>
<thead>
<tr>
<th>Chart</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>For every 5 units sold, $2 is earned.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation</th>
<th>Words</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Multistep Problems

Different Context, Same Models and Strategies

The jacket Keri wants to buy is 20% off this week. If she buys the jacket this week she will have to pay $140. How much would she have had to pay if she bought the jacket last week? Now, if she wants to wait until next week to buy the jacket, the $140 sale price will increase by 20%. How much will she have to pay next week?
Revisit Section 2 Big Questions

- What types of problems will your students need to solve in order to solve problems with ratios and proportions at a conceptual level?

- How do the instructional strategies that you choose to use help your students develop a conceptual understanding of ratios and proportions?
Reflection: Will Do

To help your students develop a conceptual understanding of ratio and proportions you will do the following:

1. 

2. 

3. 

4. 

5. 

6.
Connections to the Geometry Progression

Similar Triangle Applications
Try to solve each problem in two ways—use ratios within the triangles, and use ratios between the triangles.

1. At the airport, there is a flag pole. At a certain time each day, the shadow of the flagpole measures 9.7 m, and a man who is 170 cm tall has a shadow that measures 191 cm.

How high is the flagpole?
Remember to convert measurements to the same units.

2. Directly outside your hotel window, there is a very tall palm tree. You have heard that a hurricane may be coming, and you are concerned that the tree will fall down and smash your window. You make some direct measurements to help you determine whether you are in danger.

(America’s Choice 2009a, 64–65)
A stick measuring 1 yard has a shadow of 2.5 ft at the same time the tree has shadow of 29 ft. The distance between the base of the tree and the hotels is 12 yards. Are you safe?

1. How do these problems extend students’ understanding of ratios and proportionality?

2. How do these tasks support simultaneous development of problem solving skills and conceptual understanding?

3. What misconceptions or confusions might students bring to this problem set?

(America’s Choice 2009a, 64–65)
CCSSM Connections Search

Use the space below to make as many connections as possible to the Ratios and Proportional Relationships standards.

<table>
<thead>
<tr>
<th>Ratios and Proportional Relationship Standards</th>
<th>Connected Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Revisit Section 3 Big Questions

• How can your students’ understanding of ratios and proportions impact the teaching and learning of standards in other content areas?

________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________

• How will you help your students make connections between ratios and proportions and topics that they will study when you address other standards?

________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________
________________________________________________________________________________________
Section 4: Planning Lessons with Ratios and Proportional Relationships

Classroom Resources Brainstorm
What resources can you provide students to help them become successful with solving problems involving ratios and proportional relationships?

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________
Template for Success

Four Steps to Planning Effective Mathematical Instruction

When you plan a mathematics lesson, you start by selecting a set of problems or tasks for students that address a particular standard or cluster (depending on the lesson). You have the mathematics goal of the lesson in mind from the start of the planning process and, hopefully, you have also thought about the questions, “How will I know that students have met this goal? What evidence will I see in their work to support that claim?” Unfortunately, mathematics lessons don’t always go as planned. One way that you can help to ensure that your lessons will go as planned is by planning for an effective mathematical discussion at the end of the class.

In addition to the Standards for Mathematical Content, you must also be mindful of the Standards for Mathematical Practice. As you begin the planning process, you must consider which of the mathematical practices will be highlighted in the lesson and keep those practices in mind as you think about the mathematical discussions and experiences that you want to happen as a result of the lesson.

Good mathematical discussions do not just happen. They are the result of careful thought and planning. The goal is to keep the discussion focused on the mathematics of the lesson. The four-step process outlined below is designed with that goal in mind.

1. Work the problem set or tasks yourself! Think about all the different (correct) solutions that students might produce. For example, if the problems ask students to compare two ratios, you might naturally think of finding common denominators, but students could find decimal equivalents, use benchmark numbers, or cross multiply instead.

2. Think about the likely misconceptions and missteps. Group them with the correct solution methods that would result if the student had not taken the “misstep.”

3. Once you have an idea about the different approaches and missteps that students might take, revisit the question, “What is the important mathematics that I want students to understand after working these problems or tasks?” Ask yourself whether these problems or tasks (as they are presently worded), will result in student work that brings this mathematics to the forefront.

   a. Look back at the possible solution approaches. Is there any way that students might solve these problems correctly that will not support the mathematics of the lesson? Thinking back to the problems that ask students to compare two ratios, if the goal of the lesson is to have students understand that the procedure for finding common denominators works because of the identity property of multiplication but all of the students find decimal equivalents, it will be difficult to bring the mathematical “punch line” back to the desired end!
b. Will the problems work as they are written, or do you need to think about rewording, changing the directions, or even finding a different task or problem set? Yes, unfortunately, after all this work, sometimes you need to rethink the assignment at this point in the process.

4. Once you are happy with the problem set or tasks, answer the question, “How will I react if students do x, y, or z?” for each of the approaches or missteps you identified in Steps 1 and 2.

   a. What questions will you ask?
   b. How will you lead the discussion so that the mathematics is clarified?
   c. What if you do not see the mathematics that you want to emphasize in the student work? Will you redirect student efforts, or can you ask a student to take a particular approach?

This sounds overwhelming when you first start, but with practice, it becomes automatic to think this way. And, once you have done these things, you basically have your lesson planned!
Problem or Task

Before you begin:
What is the important mathematics that you want students to understand after completing the problem(s) or task(s)?

What Standards for Mathematical Practice do you want to highlight?

Will the problem(s) or task(s) result in student work that will bring the mathematics to the forefront? If not, what adjustments need to be made?

Work the problem(s) yourself! Identify possible (correct) solution methods.

Identify likely misconceptions related to correct solution methods.

How will you react if students do this?
Revisit Section 4 Big Questions

• What key elements must you be mindful of when you plan lessons that involve ratios and proportional relationships?

• How will your lesson-planning process change to incorporate these key elements?
Reflection and Closing

Final Reflection: Taking Action

Create three statements that describe actions that you will take in your classroom based on today’s workshop.

1. _________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________

2. _________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________

3. _________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
   __________________________________________________________________
References


Using your smart phone, tablet, or laptop Internet browser, open the Pearson Customer Satisfaction Survey or scan the QR code to open the link for a very brief survey.

http://www.pearson.com/makethegrade

Please select these answers for the following survey questions:

Q: Please select the product or service for which you are completing the survey.  [Choose: Professional Development]

Q: How did you hear of this survey?  [Choose: Workshop]

Q: Your Workshop ID is: __________________________

(to be provided by your facilitator)